A ϕ-Competitive Algorithm for Scheduling Packets with Deadlines

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University of Warwick

Joint work with **Marek Chrobak** (UC Riverside),
Łukasz Jeż (Wrocław Univ.), and
Jiří Sgall (Charles Univ., Prague).

SODA'19, January 6

Online Packet Scheduling with Deadlines

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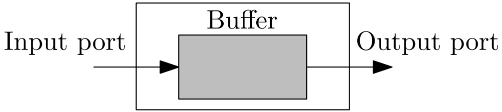

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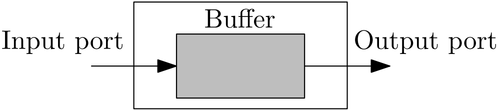

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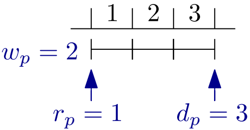

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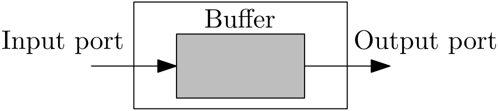

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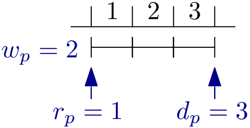

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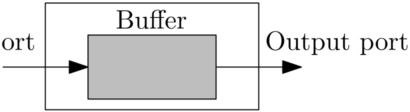

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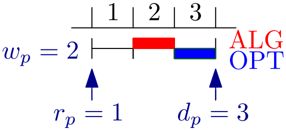

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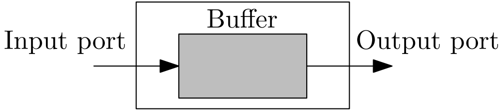

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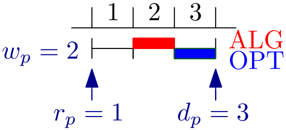

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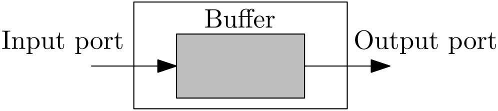
Network switch: 

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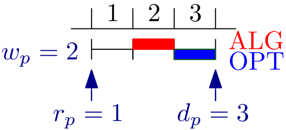

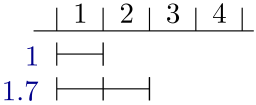

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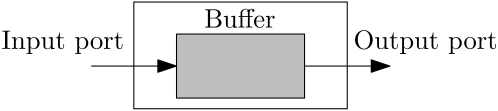

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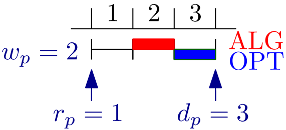

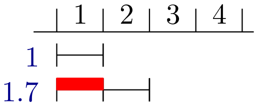

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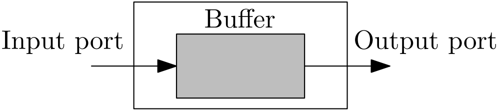

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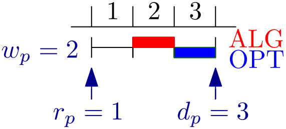

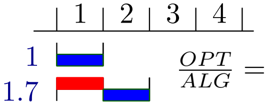

$$\frac{OPT}{ALG} = \frac{1.7+1}{1.7} \approx 1.59$$

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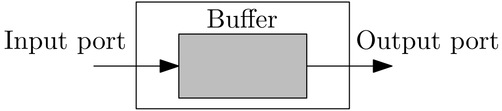
Network switch: 

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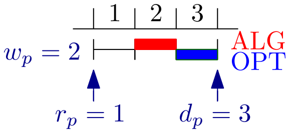

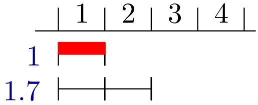

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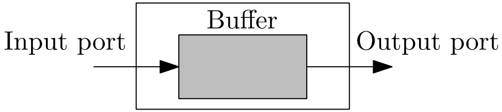

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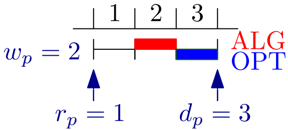

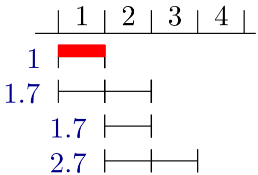

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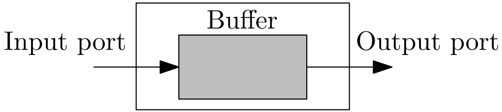

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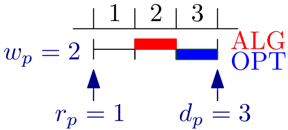

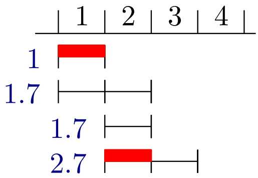

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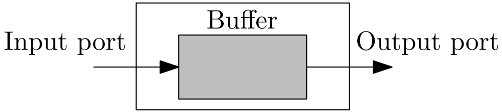

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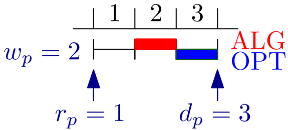

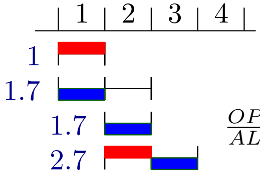

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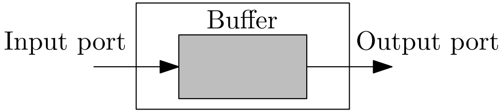

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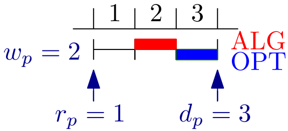

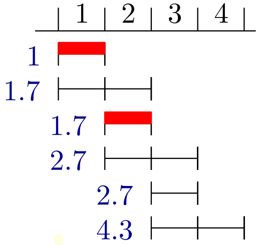

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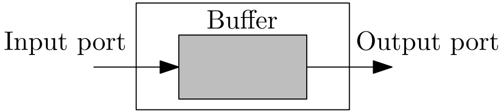

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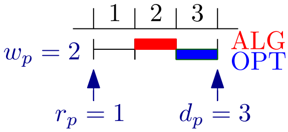

Scheduling problem $1|online, r_j, p_j = 1| \sum w_j(1 - U_j)$
A.k.a. Buffer Management in QoS Switches

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$$\mathrm{OPT}(I) \leq R \cdot \mathrm{ALG}(I)$$

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- Game: the algorithm vs. an adversary 
  - The adversary decides on further input to maximize OPT/ALG

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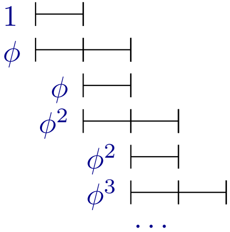

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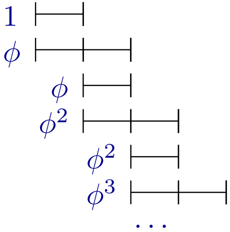

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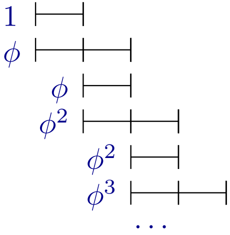

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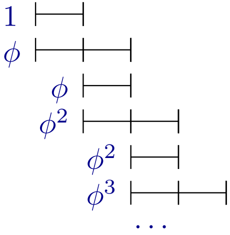

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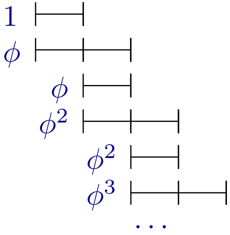

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- Scheduled plans (a.k.a. provisional schedules) used already by [Li *et al.* '05, Li *et al.* '07, Englert & Westermann '07]

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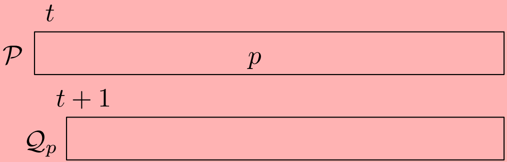

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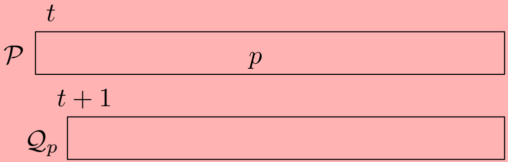

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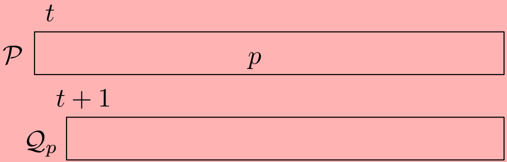

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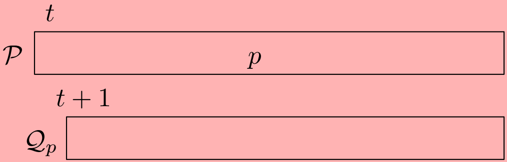

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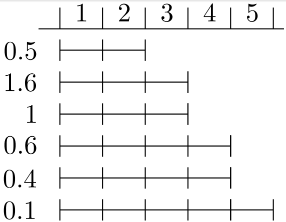

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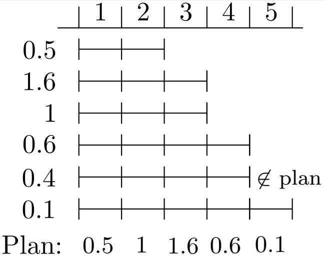

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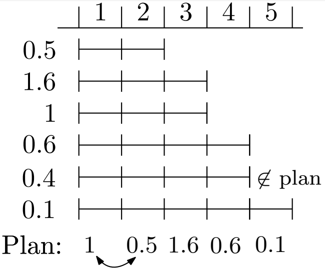

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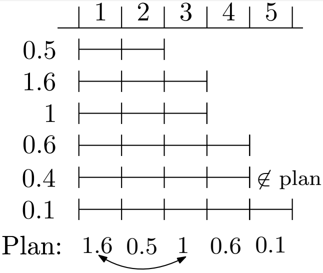

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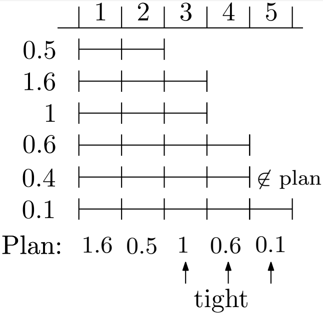

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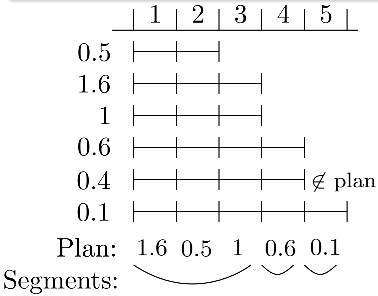

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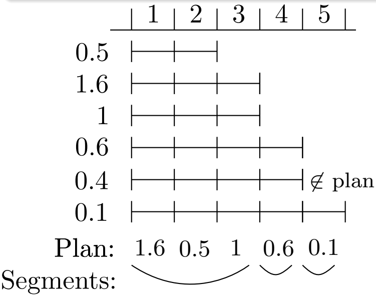

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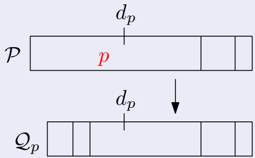

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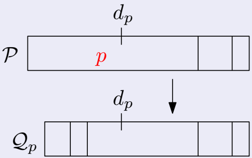

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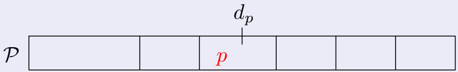

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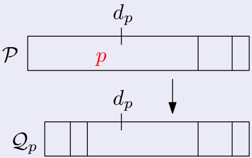

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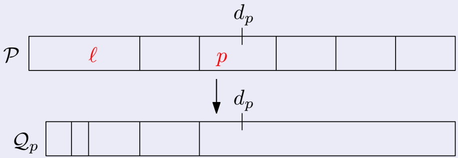

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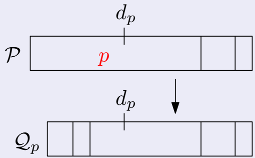

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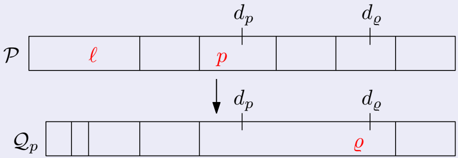

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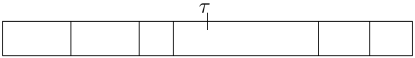

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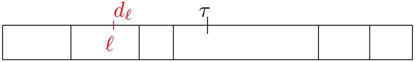

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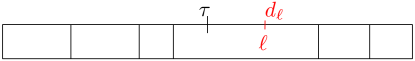

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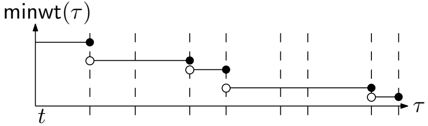

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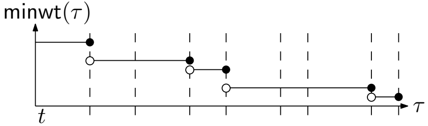

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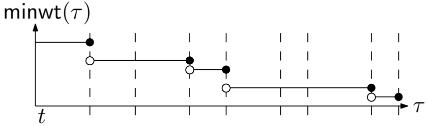

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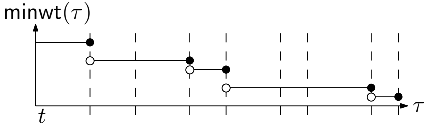

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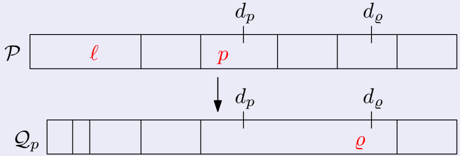

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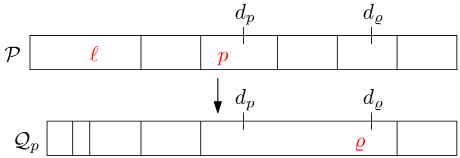

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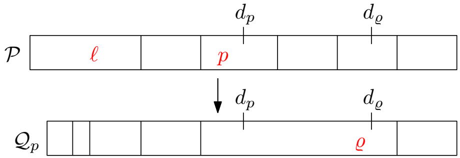

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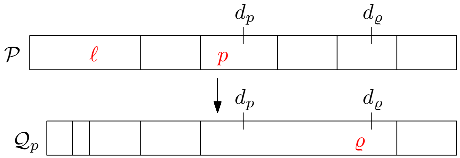

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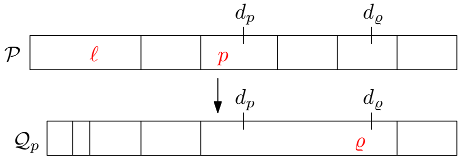

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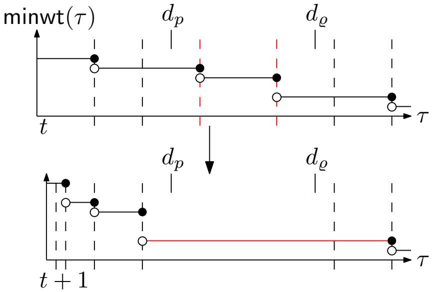

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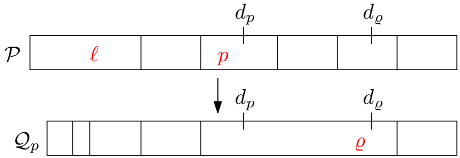

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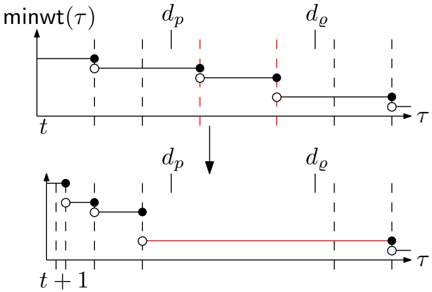

- $\Rightarrow$ avoid merging segments

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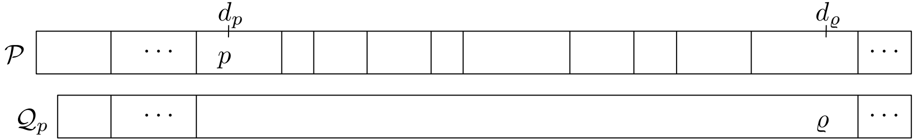

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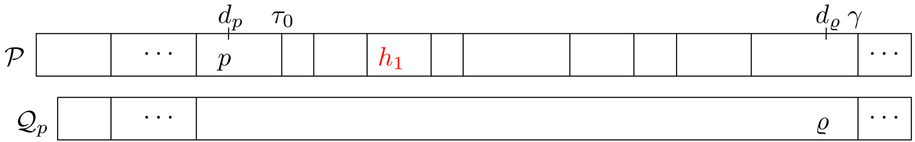

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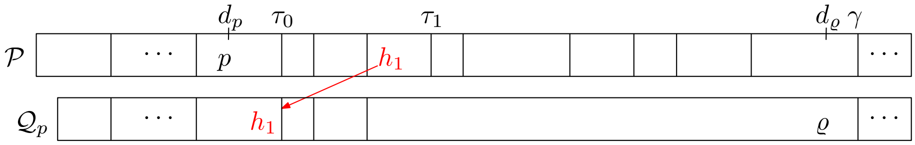

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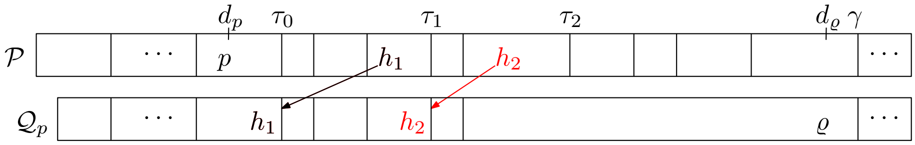

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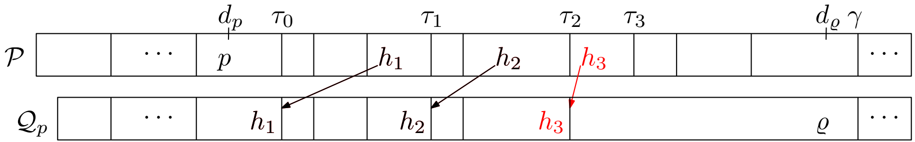

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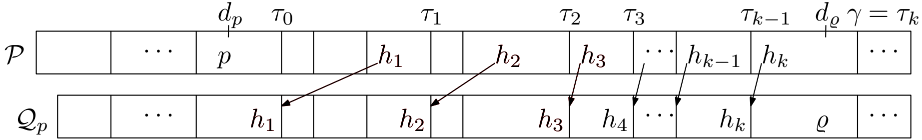

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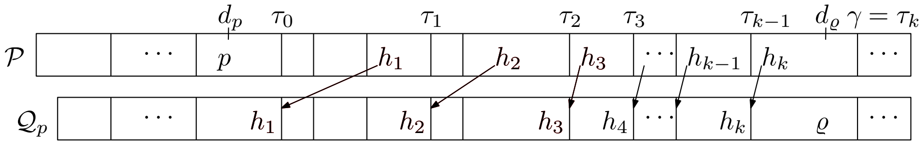

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Invariant

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