On Packet Scheduling with Adversarial Jamming and Speedup

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Goal of this talk

Simple online scheduling model with an open problem.

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Outline

- Model
- Algorithm
- Local analysis
- Non-local analysis
- Lower bounds

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[Anta, Georgiou, Kowalski, Widmer, Zavou '13], [Jurdzinski, Kowalski, Loryś '14]

- Packets of sizes $\ell_1 < \cdots < \ell_k$, released over time, no deadlines
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Easy

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buffer: $n\times$ 1 $2\times$ 1.9

ALG:

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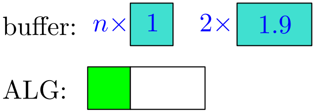

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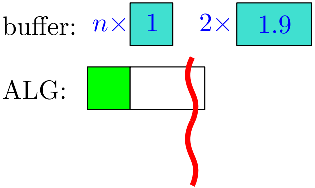

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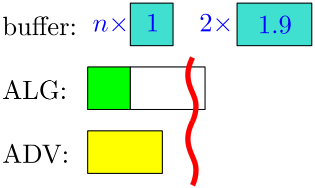

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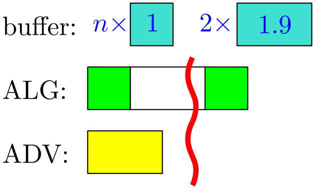

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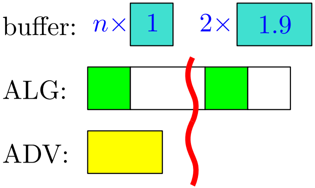

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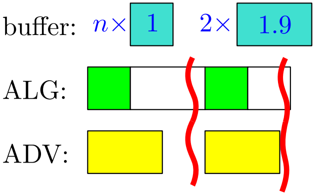

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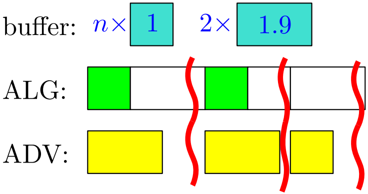

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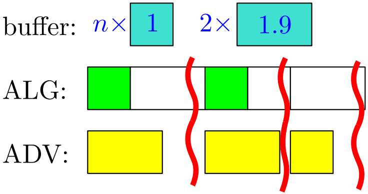

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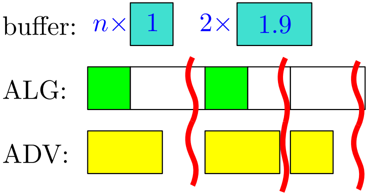

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We focus on deterministic algorithms only.

Previous work [Anta et al.], [Jurdzinski et al.]

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Speedup for 1-competitiveness on general instances

- a lower bound of 2 [Anta et al. '15]
- but no good algorithm

Speedup s = ALG needs time only ℓ/s to send a packet of size ℓ

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Lower bounds

No 1-competitive deterministic algorithm with speedup $s < \phi + 1 \approx 2.618$

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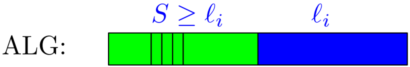

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ALG:

Phase ends when:

- fault occurs,
- no packet is pending, or
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buffer: $1\times\frac{1}{2}$ 4×1 1×4 8×5

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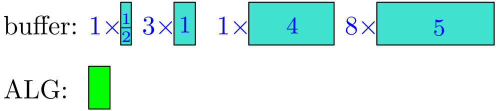

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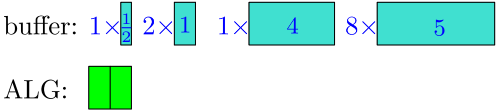

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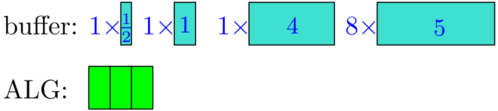

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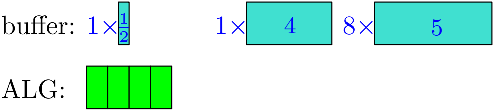

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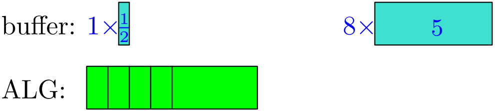

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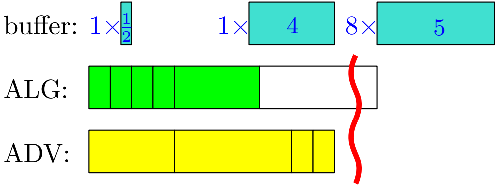

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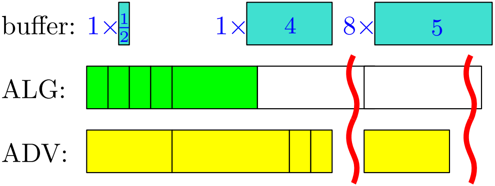

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Nice properties of the new algorithm

- no unnecessary idle time
- the same algorithm for all speeds
- no need to know the speed and packet sizes in advance

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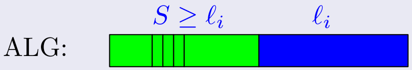
Observation: In each phase

- If the 1^{st} packet completes: size of completed packets $\geq \frac{1}{2}$ of length of the phase.



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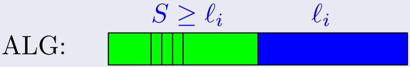

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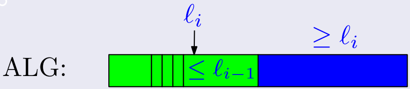

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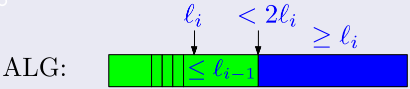

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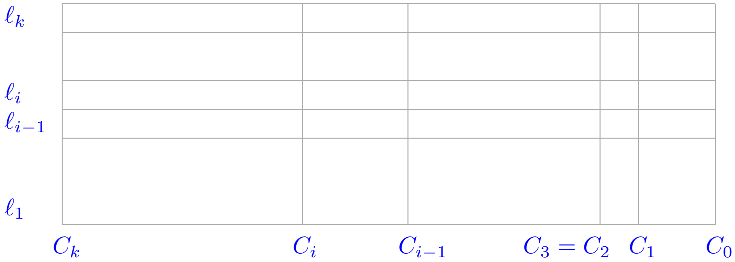

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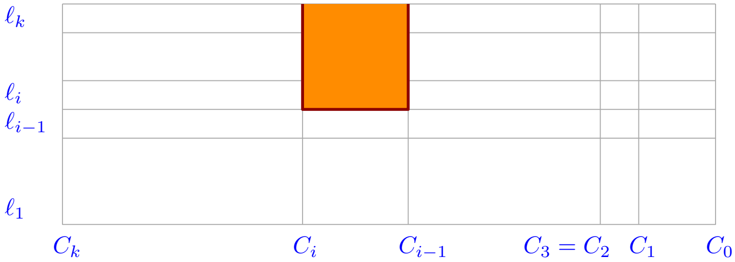

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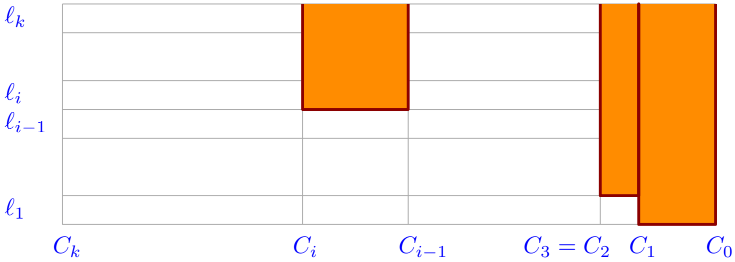

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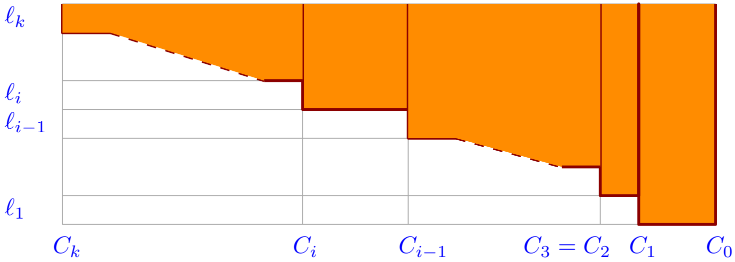

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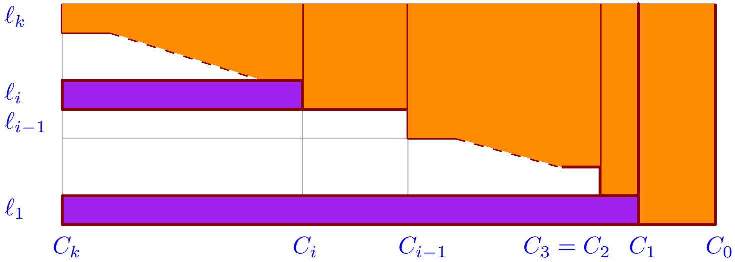

Proof method (for 1-competitiveness)

For each phase within $(C_i, C_{i-1}]$, show that the total size of long packets ($\geq \ell_i$) completed by ALG is at least that of ADV

Local analysis – The method

For each size ℓ_i: critical time C_i.

Critical times $C_k \leq C_{k-1} \leq \cdots \leq C_1$ satisfy:

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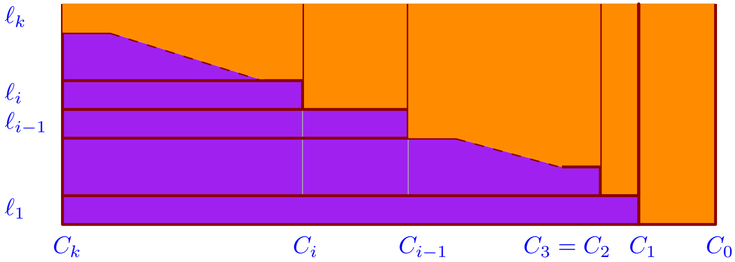

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Local analysis – General instances, speedup 6

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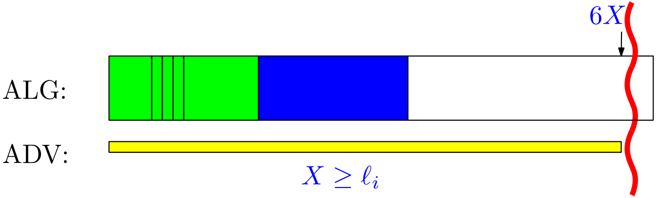

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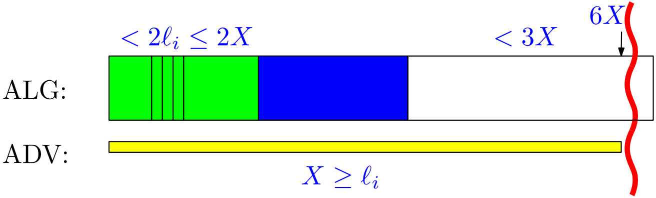

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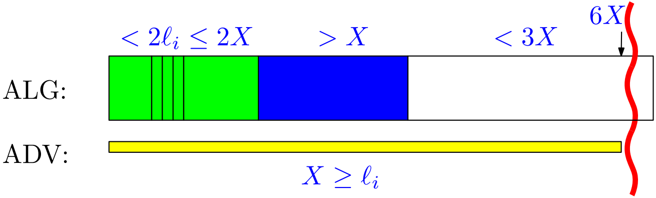

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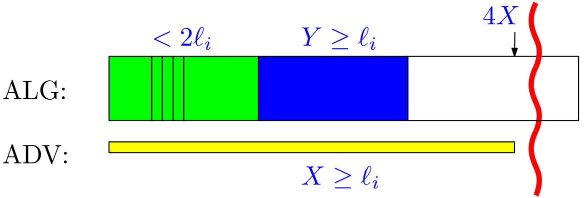

- Y completed as $Y < 2\ell_i$
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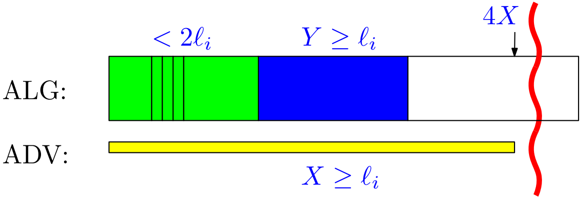

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General instances, speedup 4

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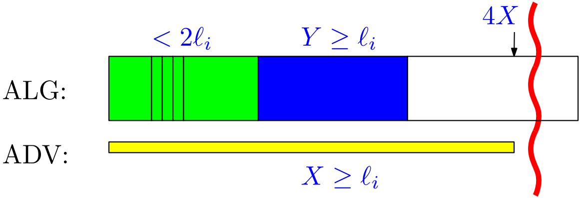

- Y completed as $Y < 2\ell_i$
  - Does not hold for speedup below 4
- It may happen that $X = \ell_{i+1} > \ell_i = Y$ ...
  ... but only if no packet of size ℓ_{i+1} is pending.

General instances, speedup 4

Redefine critical times: C'_i satisfy:

- almost no packets of size ℓ_i pending just before C'_i,
- a packet of size ℓ_i is always pending all the time after C'_i.

We may have e.g. $C'_4 > C'_1 > C'_2 = C'_5 > C'_3$.

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Focus on packets of size ℓ_i

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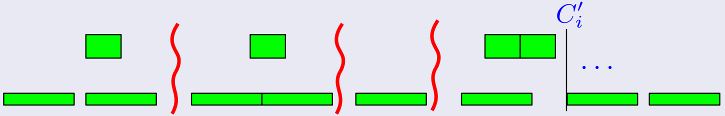

General instances, speedup 4

Redefine critical times: C_i' satisfy:

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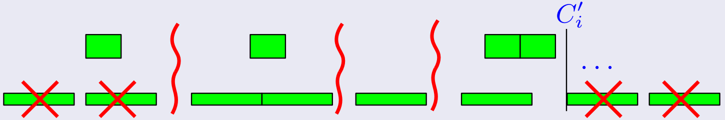

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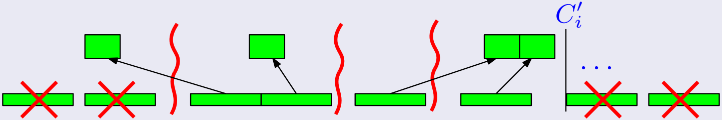

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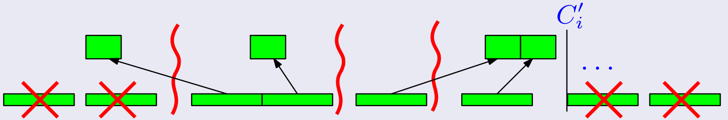

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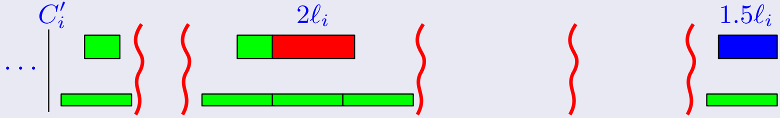

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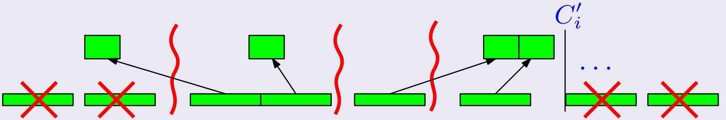

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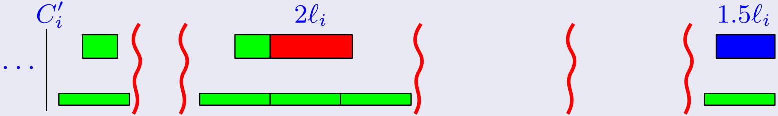

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General instances, speedup 4

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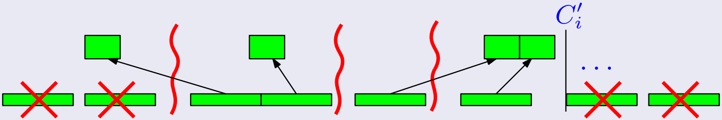

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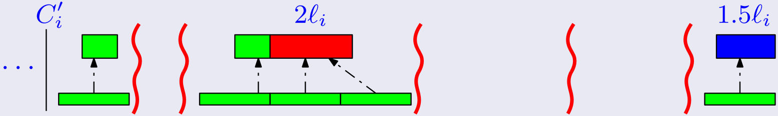

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General instances, speedup 4

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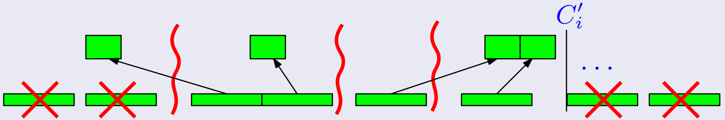

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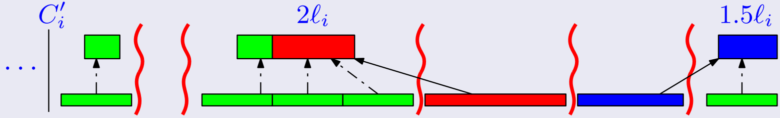

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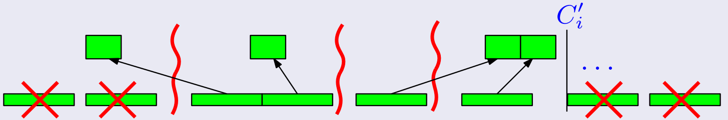

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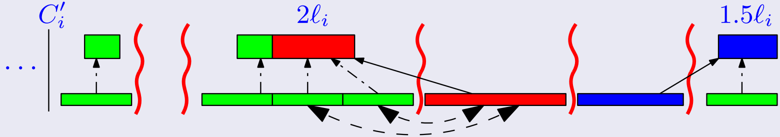

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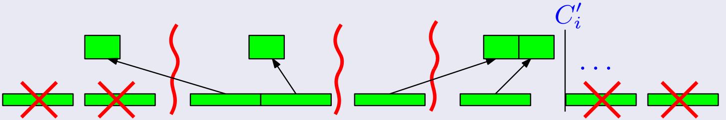

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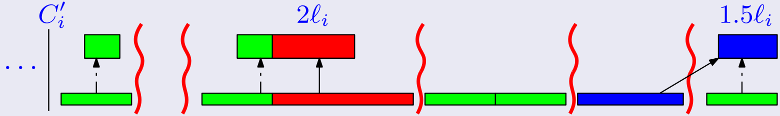

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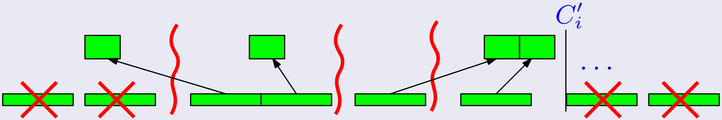

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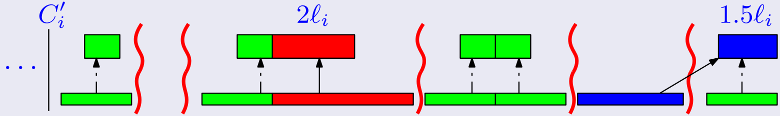

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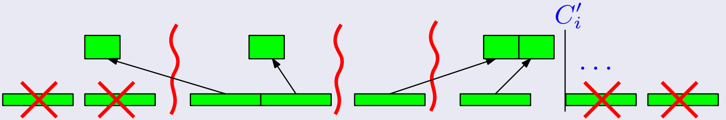

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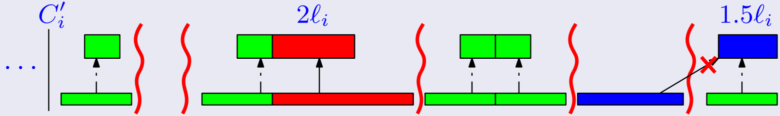

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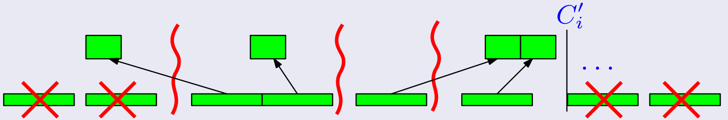

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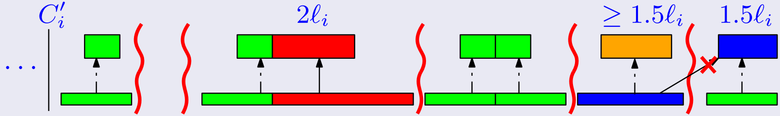

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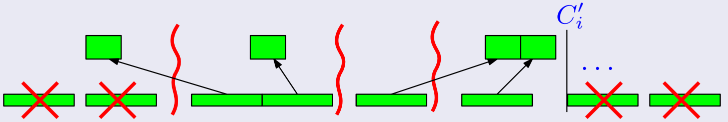

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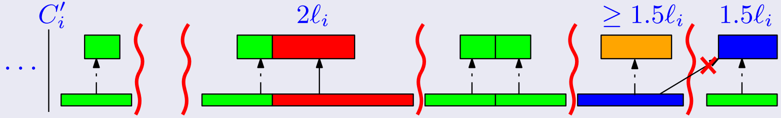

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+ quite a lot of technical work (e.g., phases in which ALG completes no packet)

Lower bound – Speed below 2

No deterministic 1-competitive algorithm with speedup < 2

Input

- packet sizes 1 and ℓ, all packets arrive at time 0
- $\#$ of small packets $\gg$ $\#$ of ℓ-packets

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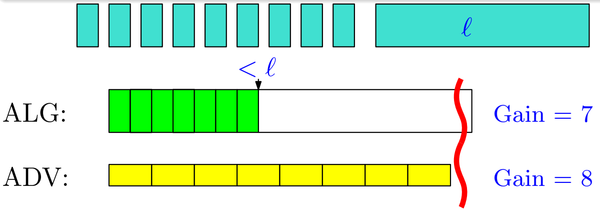

Adversary strategy in each phase

- If ALG starts ℓ soon, interrupt it and finish more small packets than ALG.

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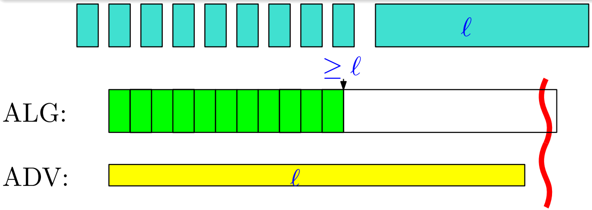

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- Otherwise finish ℓ and make a fault.

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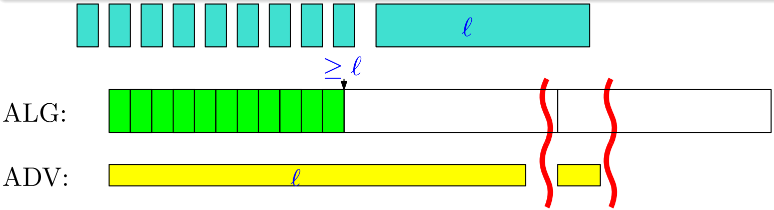

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Lower bound – Speed below $\phi + 1 \approx 2.618$

- packet sizes ε, 1, ϕ, ϕ^2, $\dots$, ϕ^k; all arrive at time 0,
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Lower bound – Speed below $\phi + 1 \approx 2.618$

- packet sizes ε, 1, ϕ, ϕ^2, . . . , ϕ^k; all arrive at time 0,
- # of smaller packets $\gg$ # of bigger packets,
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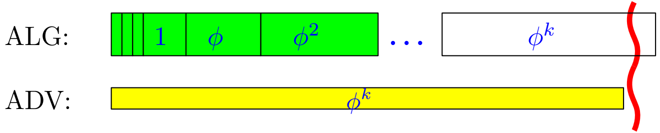

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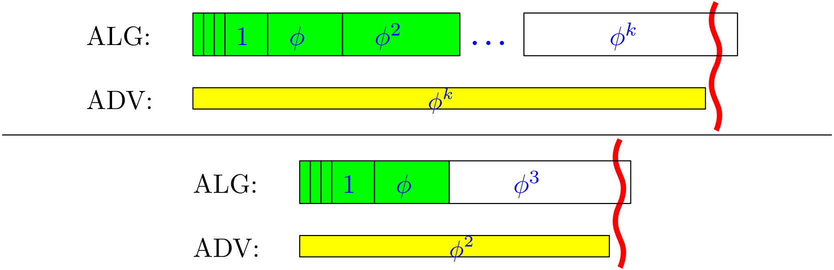

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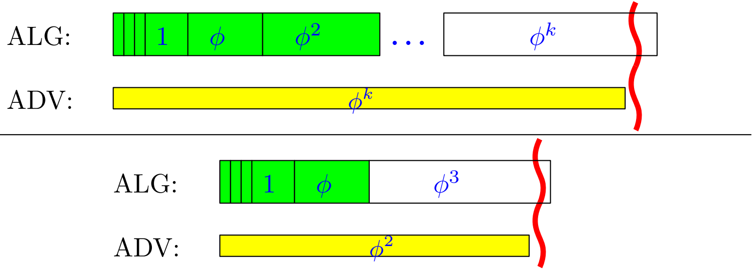

\+ two other similar cases
\+ when ADV completes all packets ϕ^i, then it completes packets $< \phi^i$ preventing ALG to finish a packet $\geq \phi^i$

Open Problems (for the way back home)

- 1-competitive algorithm with speedup $s < 4$

Open Problems (for the way back home)

- 1-competitive algorithm with speedup $s < 4$
  - Or a lower bound better than $\phi + 1$
  - Do release times change the required speedup?

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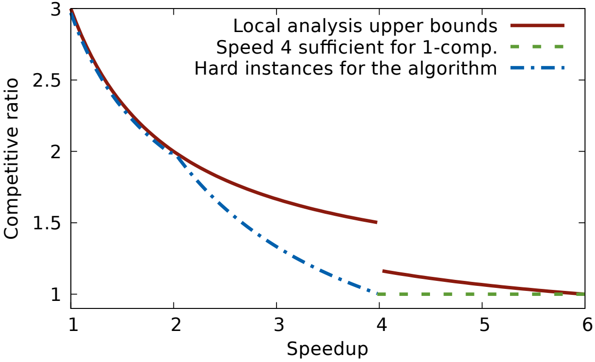

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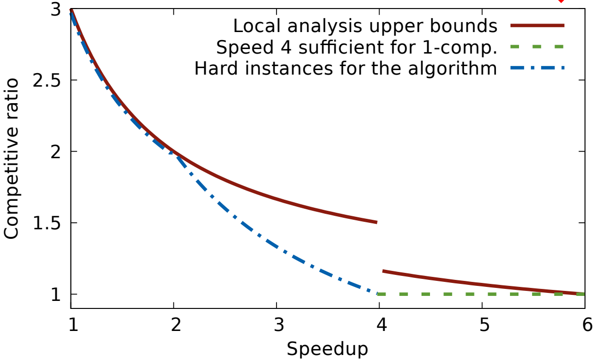
Thank You!



Local analysis results for special cases

Divisible instances (ℓ_i divides ℓ_{i+1} for each i):

- Our algorithm is 1-competitive with speedup 2.5

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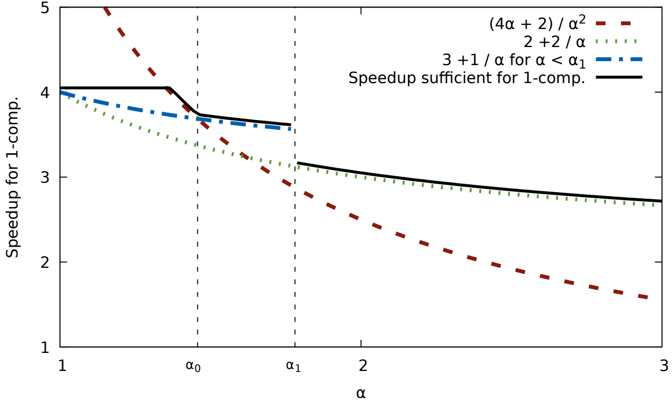
Well-separated instances:

- $\ell_{i+1} \geq \alpha\ell_i$ for some parameter $\alpha > 1$
- Our algorithm is 1-competitive with speedup S_α:



Algorithm for divisible instances

Start phase: Run packet of the largest size ℓ_j such that $P^{<j} < \ell_j$,
$P^{<j}$: total size of pending packets smaller than ℓ_j.

Regular step: Run ... largest ℓ_j such that $\ell_j \leq S$ and ℓ_j divides S,
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Results using local analysis

- 2-competitive (optimal),
- 1-competitive with speedup 2 (optimal),
- both algorithmic results also done by [Jurdzinski et al.]