Probabilistic techniques - tutorials
Classwork 2 – Linearity of expectation

1. Compute the expected number of fixed points of a random permutation on $[n]$.

2. Show that there is a two coloring of the edges of K_n with at most $\binom{n}{a}2^{1-\binom{a}{2}}$ monochromatic K_a.

3. Show that there is a two coloring of edges of $K_{n,m}$ with at most $\binom{n}{a}\binom{m}{b}2^{1-ab}$ monochromatic $K_{a,b}$.

4. Let $A, B \in \binom{n}{k}$ be chosen independently uniformly at random. Compute $\mathbb{E}[|A \cap B|]$.

5. Let M be an $n \times n$ matrix with entries uniformly independent chosen from $\{-1, 1\}$. Determine $\mathbb{E}[\det(M)]$.

6. Let $n \geq 2$, $H = (V, E)$ an n-uniform hypergraph with $|E| = 4^{n-1}$ edges. Show that there is a coloring of V by four colors such that no edge is monochromatic.