

1. Show that VERTEX COVER is FPT with parameter treewidth. Meaning that given $k \in \mathbb{N}$, we can in time $f(t)n^{\mathcal{O}(1)}$ determine whether an input graph G with $t = \text{tw}(G)$ has a vertex cover of size at most k for some computable function f .

Can you also do it in time $c^t n^{\mathcal{O}(1)}$ for some constant c ?

2. In the PLANAR VERTEX COVER problem, we are given a planar graph G and an integer k . The goal is to determine whether it has a vertex cover of size at most k .

Can you do it in time $2^{\mathcal{O}(\sqrt{k})} n^{\mathcal{O}(1)}$ for some constant?

3. Show that HAMILTONIAN CYCLE is FPT with parameter treewidth.

Hint. In the dynamic program, you will need more than just the information from the subtree.

4. Show that STEINER TREE is FPT with parameter treewidth.