1. For $Q||C_{\max}$ and $R||C_{\max}$, we define $p_{\max}$ to be the largest processing time on input. Prove that $Q||C_{\max}$ is FPT with parameter $p_{\max}$ and $R||C_{\max}$ is FPT with parameter $p_{\max} + \tau$.

   Hint. n-fold integer programming.

2. In the Closest String problem, we are given k strings $s_1, \ldots, s_k$ of length n from an alphabet of size ℓ, and an integer d. The goal is to find a string s such that its Hamming distance from each of $s_1, \ldots, s_k$ is at most d where Hamming distance between two strings is the number of indices where their letters do not match.

   We want to show that Closest String is FPT with parameter k. We can view the input as a matrix with k rows and n columns.

   a) First show that we can bound $n \leq k^k$, i.e., that the number of columns can be upper-bounded. It will be useful to show that we can bound $\ell \leq k$. This shows that there are at most k^k different *types* of columns on input.
   b) If we write the output string s under the matrix, then we can freely permute the columns of this $(k+1) \times n$ matrix and this will not affect the Hamming distance of s and $s_1, \ldots, s_k$. So in a sense, it does not matter where a column of each type is, only to which output character it maps.
   c) Using this fact, write an integer program and conclude that you have a double exponential algorithm from Lenstra's algorithm.
   d) Observe that your integer program can be rewritten as an n-fold. What are the consequences?

3. Show that Vertex Cover is FPT with parameter treewidth. Meaning that given $k \in \mathbb{N}$, we can in time $f(t)n^{\mathcal{O}(1)}$ determine whether an input graph G with $t = \operatorname{tw}(G)$ has a vertex cover of size at most k for some computable function f.

   Can you also do it in time $c^t n^{\mathcal{O}(1)}$ for some constant c?

4. In the Planar Vertex Cover problem, we are given a planar graph G and an integer k. The goal is to determine whether it has a vertex cover of size at most k.

   Can you do it in time $2^{\mathcal{O}(\sqrt{k})}n^{\mathcal{O}(1)}$ for some constant?

5. Show that Hamiltonian Cycle is FPT with parameter treewidth.

   Hint. In the dynamic program, you will need more than just the information from the subtree.