1. Listen to me explain what *scheduling problems* are.

   Glossary:

   - There are n *jobs* and m *machnes*.
   - Jobs are defined by their *processing times* p_{ij} for $i \in [1, m]$ and $j \in [1, n]$ which indicates how long does job j run on machine i.
   - A *schedule* is an assignment of jobs to machines.
   - The *makespan* of a schedule is the last completion time over all machines.

2. In Scheduling on Identical Parallel Machines, commonly known as $P||C_{\max}$, we have τ different job types which are only distinguished by their running times $p_1, \dots, p_\tau$ where p_i is the processing type of job of type i on any of the m machines, and their multiplicities $n_1, \dots, n_\tau$. Meaning that there are n_i jobs of type i, and $\sum_{k=1}^{\tau} n_k = n$ jobs in total. The goal is to find a schedule of minimum makespan.

   Give an integer programming formulation of $P||C_{\max}$ which has size bounded by τ and m. So the number of variables and constraints in the program should be bounded by a function τ and m. (And numbers in the program should be polynomial in the input size and a function of τ and m. This is to prevent you from somehow encoding the entire problem into a one variable of the program.) How small of a formulation can you find?

3. In the lecture, you saw an integer program for Graph Coloring on graphs of bounded neighborhood diversity w. What you have not seen, though, is how to recover a coloring from the solution of the integer program. That is your task now.

   Recall that T is the *type graph* of graph G. Meaning that each vertex of T is a set in the optimal neighborhood partition of G. By V_u where $u \in V(T)$ we denote vertices whose type corresponds to u. A vertex $u \in V(T)$ has a self loop iff the V_u is a clique. And $u, v \in V(T)$ have an edge iff V_u and V_v form a complete bipartite graph. (Btw. how many vertices does T have?)

   Let $\mathcal{I}$ be the set of all independent sets of T, not necessarily maximal ones. The integer program for coloring was the following. We have a variable x_I for every $I \in \mathcal{I}$. The formulation is as follows

$$\min \sum_{I \in \mathcal{I}} x_I$$
$$\text{such that} \sum_{I : v \in I} x_I = |V_v| \quad \forall v \in V(T)$$

   Question. Suppose I give you an optimum solution of this integer program with $\sum_{I \in \mathcal{I}} x_I = c$. Can you produce a valid c-coloring of G?

4. In the Precoloring Extension problem, we are given a graph G where some vertices are precolored, and an integer c. The goal is to find a c-coloring of graph G which respects the precoloring which you can assume is a proper coloring.

   Give an FPT algorithm for parameter neighbourhood diversity for Precoloring Extension.