1. What is the treewidth of the following graphs? If you are struggling, try to give just an upper bound.

   a) A complete graph K_n.

   b) A complete bipartite graph $K_{n,m}$.

   c) A forest.

   d) A cycle C_n.

   e) A cube Q_3.

   f) A $m \times n$ grid $\boxplus_{m,n}$.

2. Show that for every minor H of graph G, we have $\mathrm{tw}(H) \leq \mathrm{tw}(G)$.

3. Prove the following lemma. It says something to the effect that given a tree decomposition, we can always find a nice tree decomposition in polynomial time.

   If a graph G admits a tree decomposition of width at most k, then it also admits a *nice tree decomposition* of width at most k. Moreover given a tree decomposition $\mathcal{T} = (T, \{X_t\}_{t \in V(T)})$ of G of width at most k, one can in time $\mathcal{O}(k^2 \cdot \max\{|V(T)|, |V(G)|\})$ compute a nice tree decomposition of G of width at most k that has at most $\mathcal{O}(k|V(G)|)$ nodes.

4. How can subdividing an edge of a graph G change its treewidth? Can in increase or decrease?

5. Show that the treewidth of a graph G is equal to the maximum treewidth of its biconnected components.