1. Let $M(G)$ be the size of a maximum matching of graph G.

   Give an algorithm for Vertex Cover Above Matching running in time $4^{k-M(G)}n^{\mathcal{O}(1)}$. Meaning that the parameter is not the size of the solution but the "excess" above the lower bound given by the maximum matching.

   Variable Deletion Almost 2-SAT
   Input: 2-CNF formula $\varphi = C_1 \wedge \cdots \wedge C_m$ on variables $x_1, \ldots, x_n$, parameter $k \in \mathbb{N}$.
   Output: Is there a subset of variables $\mathcal{X} \subseteq \{x_i\}_{i=1}^n$ of size at most k so that φ without clauses that contain a variable of $\mathcal{X}$ is satisfiable?

2. Show an algorithm for Variable Deletion Almost 2-SAT running in time $4^k n^{\mathcal{O}(1)}$.

   Hint: Proceed as in the algorithm for Odd Cycle Transversal from the lecture.

   Almost 2-SAT
   Input: 2-CNF formula $\varphi = C_1 \wedge \cdots \wedge C_m$ on variables $x_1, \ldots, x_n$, parameter $k \in \mathbb{N}$.
   Output: Is there a subset of clauses $\mathcal{C} \subseteq \{C_i\}_{i=1}^m$ of size at most k so that $\varphi \setminus \mathcal{C}$ is satisfiable?

3. Show an algorithm for Almost 2-SAT running in time $4^k n^{\mathcal{O}(1)}$.

The exercises above might be too easy. So here are some extra.

Definition. Let $\mathcal{A}$ be a family of sets (without duplicates) over a universe $\mathcal{U}$. A *sunflower* with k *petals* and a core Y is a collection of sets $S_1, \ldots, S_k \in \mathcal{A}$ such that $S_i \cap S_j = Y$ for all $i \neq j$; the sets $S_i \setminus Y$ are *petals* and we require none of them to be empty. Note that a family of pairwise disjoint sets is a sunflower (with an empty core).

Sunflower lemma. Let $\mathcal{A}$ be a family of sets (without duplicates) over a universe $\mathcal{U}$, such that each set in $\mathcal{A}$ has cardinality exactly d. If $|\mathcal{A}| > d!(k-1)d$, then $\mathcal{A}$ contains a sunflower with k petals and such a sunflower can be computed in time polynomial in $|\mathcal{A}|$, $|\mathcal{U}|$, k.
d-Hitting Set
Input: A family of sets $\mathcal{A}$ (without duplicates) over a universe $\mathcal{U}$, such that each set in $\mathcal{A}$ has cardinality exactly d, integer $k \in \mathbb{N}$.
Output: Is the a subset $H \subseteq \mathcal{U}$ of size at most k such that every set $S \in \mathcal{A}$ contains at least one element of H?

4. Design a kernel for d-Hitting Set with at most $d!k^d d$ sets and at most $d!k^d d^2 elements$.