1. A graph G is a *cluster graph* if every connected component of G is a clique. In the CLUSTER EDITING problem, we are given as input a graph G and an integer k. The objective is to check whether one can edit (add or delete) at most k edges of G to obtain a cluster graph. That is, we look for a set $F \subseteq \binom{V}{2}$ of size at most k such that the graph $(V, (E \setminus F) \cup (F \setminus E))$ is a cluster graph.

   1. Show that a graph G is a cluster graph if and only if it does not have an induced path on three vertices.
   2. Show a kernel for CLUSTER EDITING with $\mathcal{O}(k^2)$ vertices.

2. In the MIN-ONES-2-SAT problem, we are given a 2-CNF formula φ and an integer k. The objective is to decide whether there exists a satisfying assignment for φ with at most k variables set to true. Show that MIN-ONES-2-SAT admits a polynomial kernel.

3. In the RAMSEY problem, we are given as input a graph G and a integer k, and the objective is to test whether there exists in G an independent set or a clique of size at least k. Show that RAMSEY is FPT.

4. Recall the *relaxation* of the *integer linear programming* formulation of VERTEX COVER given an input graph $G = (V, E)$. For every vertex $u \in V$ we introduce an integer variable x_u. The relaxation, is as follows.

$$\begin{aligned} \text{minimize} \quad & \sum_{u \in V} x_u \\ \text{subject to} \quad & x_u + x_v \geq 1 \text{ for every } uv \in E, \\ & x_u \geq 0 \text{ for every } u \in V, \end{aligned}$$

   Show that there exists an optimum solution that is *half-integral*, that is, a solution where every variable has one of the values $\left\{0, \frac{1}{2}, 1\right\}$.

   Hint. Take a non-half-integral solution and try to make it into a solution where fewer variables are non-half-integral.

   Unofficial homework. Think of how this implies a kernel for VERTEX COVER of size $2k$. You will find out the answer tomorrow on the lecture.