Neighborhood diversity and ILP

Exercise 1. In Scheduling on Identical Parallel Machines, commonly known as $P||C_{\max}$, we have τ different job types which are only distinguished by their running times $p_1, \dots, p_\tau$ where p_i is the processing type of job of type i on any of the m machines we schedule for, and their multiplicities $n_1, \dots, n_\tau$. Meaning that there are n_i jobs of type i, and $\sum_{k=1}^{\tau} n_k = n$ jobs in total. The goal is to find a schedule of minimum makespan.

Give an integer programming formulation of $P||C_{\max}$ which has size bounded by τ and m. So the number of variables and constraints in the program should be bounded by a function of τ and m. How small of a formulation can you find?

From your knowledge of parameterized algorithms for Integer Programming, what is the running time of the algorithm you obtained?

Exercise 2. In the k-Vertex-Disjoint Paths problem, we are given a graph G, and a set $P = \{(s_i, t_i)\}_{i=1}^{k}, s_i, t_i \in V(G)$, and the goal is to find k vertex-disjoint paths between s_i's and t_i's. The solution then is k paths $P_1, \dots, P_k$ such that the endpoints of P_i are s_i and t_i and the P_is should be vertex-disjoint.

Give an FPT algorithm for parameter neighbourhood diversity for k-Vertex-Disjoint Paths.

Exercise 3. In the Precoloring Extension problem, we are given a graph G where some vertices are precolored, and an integer c. The goal is to find a c-coloring of graph G which respects the precoloring.

Give an FPT algorithm for parameter neighbourhood diversity for Precoloring Extension.