1. Show a parameterized reduction from Multicolored Independent Set to Dominating Set.

2. Prove that Dominating Set remains W[2]-hard even if restricted to graphs excluding the star $K_{1,4}$ as an induced subgraph.

3. In the Steiner Tree problem, we are given an edge-weighted graph G, and a subset $K \subseteq V(G)$. The goal is to find a connected subgraph H of G of minimum weight such that $K \subseteq V(H)$.

   You might have seen on the lecture that if the parameter is $|K|$, then the problem is FPT (with running time $\mathcal{O}^*(3^{|K|})$. Now, we define the parameter to be $h = |V(H)| - |K|$, meaning that the parameter is the amount of vertices of $V(G) \setminus K$ we are allowed to use to build graph H. Show that Steiner Tree with parameter h is W[2]-hard.

   Note. The vertices $V(H) \setminus K$ are called *Steiner vertices* in literature.

4. Given a graph G and an integer k, the Induced Matching problem asks for an induced matching of size k, that is, k edges $x_1y_1, \ldots, x_ky_k$ such that the $2k$ endpoints are all distinct and there is no edge between $\{x_i, y_i\}$ and $\{x_j, y_j\}$ for $i \neq j$.

   Prove that Induced Matching is W[1]-hard.