1. Observe that for every clique type $V_i \in T(G)$, we need to select at least $|V_i|-1$ of its vertices into our solution. For an independent set type, we have to either select none of its vertices or all of them. This leads us to the following:

   - For every clique type $V_i \in T(G)$, decrease k (the allowed number of vertices in a VC) by $|V_i| - 1$ and update $|V_i| := 1$ in $T(G)$.
   - (Optional: apply standard VC reductions on $T(G)$.)
   - Now for each $V_i \in T(G)$, either all $|V_i|$ vertices of V_i are selected, or none are. Guess a subset of $V_{T(G)}$ as a candidate vertex cover and check whether it is valid.
   - *We get a $\mathcal{O}^*(2^{nd(G)})$ running time. The same can be achieved by at least two different branch and bound approaches.*

2. Notice that $nd(G^2) \leq nd(G)$. In both cases, we work with a modification of $T(G)$ (one only needs to be careful about which types now have self-loops).

   Now it's standard coloring (on a modified type graph).

3. It suffices to define a variable $x_v \in \{0, 1\}$ for each voter v. We are minimizing the sum $\sum_v c_v x_v$.

   For a project p, let A_p be the set of voters who want to vote *against* p. If $|A_p| \geq \frac{n}{2}$, add the constraint $\sum_{v \in A_p} x_v \geq \lceil \frac{n+1}{2} \rceil - (n - |A_p|)$.

   This is an ILP with (at most) k constraints.

4. *If we only want* XP *time, we can guess how many vertices of each type get selected, and further how the vertex capacities distribute over neighboring types.*

   Again, we don't know how to solve this in FPT *time. What follows is just brainstorming:*

   For each $V_i \in T(G)$ fix an order $v_{i,1}, v_{i,2}, \ldots, v_{i,|V_i|}$ such that $c(v_{i,a}) \geq c(v_{i,b})$ when $a < b$. Denote

   $$V_{i,j} = \{v_{i,1}, \ldots, v_{i,j}\},$$
   $$f_i(j) = \sum_{v \in V_{i,j}} c(v).$$

   The f_is are concave piecewise linear functions. If x_i vertices of V_i get selected into our vertex cover, we may assume it's the vertices V_{i,x_i}.

   What changes compared to the standard VC? Still, at least $|V_i| - 1$ vertices need to be selected from a clique type V_i. If V_i is an independent set type and for at least one of its neighbors V_j (in the type graph $T(G)$), $x_j < |V_j|$, all vertices of V_i need to be selected into a solution.

- We might want to guess which nodes of $T(G)$ aren't fully selected into the solution (i.e., guess an independent set I in $T(G)$, since we know that there cannot be neighboring independent set types that are both not selected fully).
- *This should tell us what happens with the clique types (if such a V_i is in I, $x_i = |V_i| - 1$).*
- Can we manage with a convex IP with variables $x_i \in [0, |V_i|]$ for each type i?