ITERATIVE COMPRESSION

Recap

- imagine we are solving a minimization graph problem Π w/ the following properties
  - solution is a subset of vertices (works for edges as well)
  - the solution is monotone wrt taking subgrs
    - i.e. for any $H \subseteq G$ $|opt(H)| \leq |opt(G)|$
- then we can apply the following idea
- order the vertices arbitrarily $v_1, v_2, \dots, v_n$
- let G_i be $G[\{v_1, v_2, \dots, v_i\}]$
- assume that $X_k = \{v_1, \dots, v_k\}$ is an opt sol for G_k (might have to be modified depending on problem at hand)
- for $i \leftarrow k+1, \dots, n$:
  - $Z_i = X_{i-1} \cup \{v_i\}$ # take the sol on G_{i-1} and add v_i to it
  - if $|Z_i| \leq k$: # $|Z_i| \leq k$, so we have a sol of size $\leq k$ on G_i, we are done with G_i
    - $X_i = Z_i$
    - continue
  - for $X_Z \subseteq Z_i$ # else $|Z_i| = k+1$. Now branch on all possible ways that # an opt sol on G_i can interact w/ Z_i. We are # interested because Z_i is a solution of Π, thus it # carries some struct info. Eg. Π = Vertex Cover $\Rightarrow$ $G \setminus Z$ is edgeless, # Π = FVS $\Rightarrow$ $G \setminus Z$ is a forest, ...
    - $W = X \setminus X_Z$ # W is still a solution of Π !!!

MAGIC $\left\{ \right.$
- find a solution Y to Π of size $k - |X_Z|$ in $G \setminus X_Z$ s.t. $Y \cap W = \emptyset$
  - # why $Y \cap W = \emptyset$? Because we guess X_Z to be intersection w/ opt sol in G_i $\Rightarrow$ W is out of opt sol

- usually, we just need to solve the magic part which defines the following problem

DISJOINT Π

<u>In</u>: gr G, sol S to Π, $k \in \mathbb{N}$

<u>Out</u>: find a solution Y of size $\leq k$ to problem π in G s.t. $Y \cap S = \emptyset$

- in today's tutorial, we focus on solving DISJOINT problems

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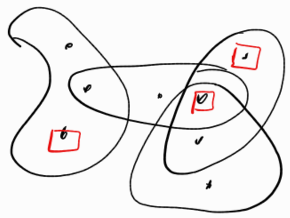
1. DISJOINT 3-HS $((U, \mathcal{S}), X, k)$



- we are given a hitting set of size $k+1$
- observe that each set has ≥ 1 elem that is undeletable due to the disj. part
- so treat each set $-X$ as an edge $\Rightarrow$ solve vertex cover

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2. in general, we are solving $(d-1)$-HS in each disj. problem
   - solvable by branching

3.

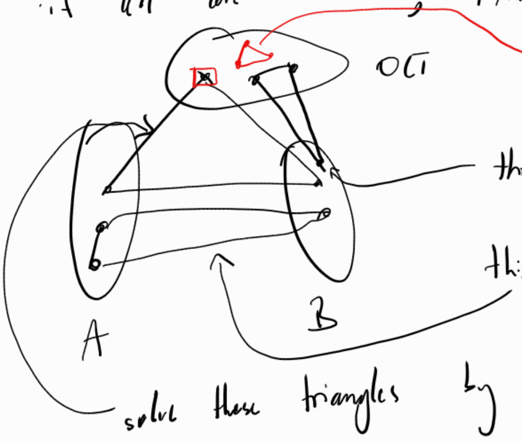
- removing an OCT makes a gr bipartite ($=$ 2-colorable)
- in the disjoint OCT problem, we have this:
- there is an OCT given to us, we need to find another one

- where does the perfectness gr. help?

Perfect gr. is bipartite $\Leftrightarrow$ it has no triangles

· find all triangles, at least one of its vertices is in OCT

· if all are FAIL



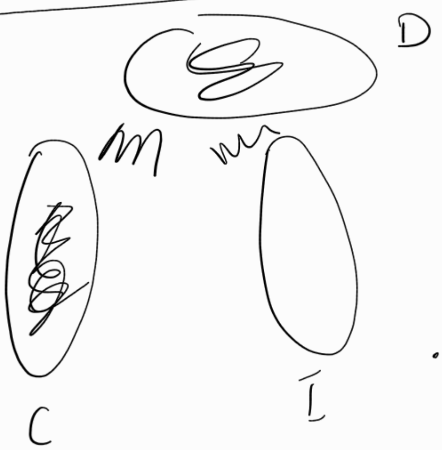
see this? FAIL

this vertex has to be in sol, reduce

this cannot happen as we have an OCT

solve these triangles by VC on bip grs

4.



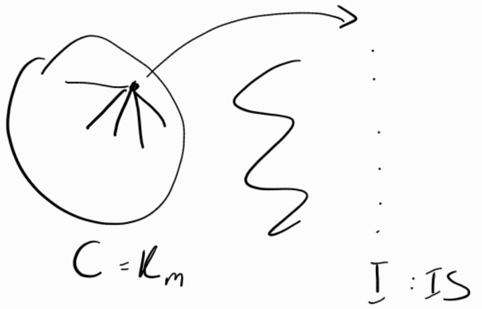
· in the disjoint problem, we have $D \subseteq V$ st. $G \setminus D$ has a partition into a clique C on IS I

· first, if $G[D]$ is not a split gr, we reject

· how to recognize this? Trust me that there exists a poly alg

- let degrees of gr be $d_1 \geq d_2 \geq \dots \geq d_n$
- let m be the largest val of i s.t. $d_i \geq i-1$
- G is a split gr iff $\sum_{i=1}^{m} d_i = m(m-1) + \sum_{i=m+1}^{n} d_i$

· if a gr. is a split gr, it can be partitioned in $\leq n^2$ ways



· ≤ 1 vertex u of C can be moved to I to preserve that $C \setminus u, I \cup \{u\}$ is a split gr. partition

· same in the other direction

→ brute force $\le n^2 \cdot n^2 = n^4$ partitions

D $\quad$ $V \setminus D$

- what does this mean? If there is a partition of V, then there is a partition of V/D as well
- but what does it look like? We don't know but we can guess which vertex of $V \setminus D$ belong to the clique of V and which to the IS (possibly none of these vertex exist as the clique/IS might be empty in the opt sol)
    - and from those choices we can complete the clique/IS as much as possible
    - and if the "uncompletable" part has size $\le k$, then that's our deletion set
- for more details, check Lemma 3 of the following paper

https://link.springer.com/article/10.1007/s00453-013-9837-5

- they get $O(n^3)$ while we get $O(n^5)$ because we only prove that there's $O(n^2)$ partitions of a split graph while they prove $O(n)$