

1. Listen to me explain what *scheduling problems* are.

Glossary:

- There are  $n$  jobs and  $m$  machines.
- Jobs are defined by their *processing times*  $p_{ij}$  for  $i \in [1, m]$  and  $j \in [1, n]$  which indicates how long does job  $j$  run on machine  $i$ .
- A *schedule* is an assignment of jobs to machines.
- The *makespan* of a schedule is the last completion time over all machines.

2. In SCHEDULING ON IDENTICAL PARALLEL MACHINES, commonly known as  $P||C_{\max}$ , we have  $\tau$  different job types which are only distinguished by their running times  $p_1, \dots, p_\tau$  where  $p_i$  is the processing time of job of type  $i$  on any of the  $m$  machines, and their multiplicities  $n_1, \dots, n_\tau$ . Meaning that there are  $n_i$  jobs of type  $i$ , and  $\sum_{k=1}^{\tau} n_k = n$  jobs in total. The goal is to find a schedule of minimum makespan.

Give an integer programming formulation of  $P||C_{\max}$  which has size bounded by  $\tau$  and  $m$ . So the number of variables and constraints in the program should be bounded by a function of  $\tau$  and  $m$ . (And numbers in the program should be polynomial in the input size and a function of  $\tau$  and  $m$ . This is to prevent you from somehow encoding the entire problem into a one variable of the program.) How small of a formulation can you find?

From your knowledge of parameterized algorithms for INTEGER PROGRAMMING, what is the running time of the algorithm you obtained?

3. In the  $k$ -VERTEX-DISJOINT PATHS problem, we are given a graph  $G$ , and a set  $P = \{(s_i, t_i)\}_{i=1}^k$ ,  $s_i, t_i \in V(G)$ , and the goal is to find  $k$  vertex disjoint paths between  $s_i$ 's and  $t_i$ 's. Meaning that the solutions is  $k$  paths  $P_1, \dots, P_k$  such that the endpoints of  $P_i$  are  $s_i$  and  $t_i$  and  $P_i$ 's should be vertex disjoint.

Give an FPT algorithm for parameter neighbourhood diversity for  $k$ -VERTEX-DISJOINT PATHS.

4. In the PRECOLORING EXTENSION problem, we are given a graph  $G$  where some vertices are precolored, and an integer  $c$ . The goal is to find a  $c$ -coloring of graph  $G$  which respects the precoloring which you can assume is a proper coloring.

Give an FPT algorithm for parameter neighbourhood diversity for PRECOLORING EXTENSION.