1. Give an algorithm for finding a minimum Vertex Cover parameterized by the treewidth $t$ of the input graph running in time $\mathcal{O}^*(f(t))$ for some computable function $f$.

   Can you also get an algorithm where $f(t) = c^t$ for some constant $c$?

2. In nice tree decompositions it it may be useful to introduce not only vertices but also edges. Let $G$ be a graph and $(T, \{X_t\}_{t\in V(T)})$ be its nice tree decomposition. An *introduce edge node* $t$ of a nice tree decomposition is labeled with an edge $uv \in E(G)$ such that $u, v \in X_t$, and with exactly one child $t'$ such that $X_t = X_{t'}$. We say that the edge $uv$ is *introduced* at node $t$. We additionally require that each edge of $E(G)$ is introduced exactly once.

   Give an algorithm that transforms a nice tree decomposition into a nice tree decomposition with introduce edge nodes in time $k^{\mathcal{O}(1)}n$. Also prove that the number of nodes is still $\mathcal{O}(kn)$. It might be useful to assume that the bag of the root node is empty.

   You can skip solving this exercise and use it as a black box.

   Usually in dynamic programming over treewidth, you only consider the state of the solution on the graph induced by vertices that lie below the bag we are considering. This approach will fail in the following two problems.

3. Give an algorithm for finding a minimum Dominating Set parameterized by the treewidth of the input graph.

   **Hint.** Let $V_t$ be the set of vertices of $G$ which appear in bags in the subtree of $T$ rooted in $t$. When dealing with a node $t$, it is not enough to consider how the solution looks like in $G[V_t]$. So in the dynamic programming table you also need to capture the fact that some of the vertices in $X_t$ might be dominated by a vertex of $V(G) \setminus V_t$.

4. Give an algorithm for Hamiltonian Cycle parameterized by the treewidth of the input graph.

   **Hint.** As in the case of Dominating Set you need information from $V(G) \setminus V_t$.