1. Using dynamic programming over subsets, obtain an algorithm for Chromatic Number on n-vertex graphs running in time $3^n n^{\mathcal{O}(1)}$.

2. For an $n \times n$ matrix A, the *permanent* of A is the value $\mathrm{perm}(A) = \sum_\sigma \Pi_{i=1}^n A_{i,\sigma(i)}$ where the sum ranges over all permutations σ of $[n]$.

   Using dynamic programming over subsets, show how to compute the permanent of a given $n \times n$ matrix in time $2^n n^{\mathcal{O}(1)}$.

3. In the Directed Feedback Arc Set, we are given a directed graph G and an integer k, and the goal is to find a subset of arcs X of size at most k such that $G \setminus X$ contains no directed cycles.

   Using dynamic programming over subsets, show that Directed Feedback Arc Set on n-vertex graphs can be solved in time $2^n n^{\mathcal{O}(1)}$.

4. Given a directed graph G, a set of terminals $K \subseteq V(G)$ and a root $r \in V(G)$, Directed Steiner Tree asks for a directed tree rooted at r such that every terminal in K is reachable from r on the tree. Obtain a $3^{|K|} n^{\mathcal{O}(1)}$-time algorithm for Directed Steiner Tree.