From now until forever we denote $[a, b] = \{c \in \mathbb{Z} \mid a \leq c \leq b\}$, and $[a] = [1, a]$.

1. In the d-Hitting Set problem, we are given a *ground set* $U = [n]$, m subsets $\mathcal{S} = \{S_i\}_{i=1}^m$ where $S_i \subseteq U$ and $|S_i| = d$, and an integer k. The goal is to find $X \subseteq U$ of size at most k such that $X \cap S_i \neq \emptyset$ for every $i \in [m]$.

   Obtain an algorithm for 3-Hitting Set running in time $\mathcal{O}^*(3^k)$ using iterative compression.

2. Obtain an algorithm for d-Hitting Set running in time $\mathcal{O}^*(d^k)$ using iterative compression.

3. An undirected graph G is called *perfect* if for every induced subgraph H of G, the size of the largest clique in H is the same as the chromatic number of H. We consider the Odd Cycle Transversal problem, restricted to perfect graphs. Show an algorithm with running time $\mathcal{O}^*(2^k)$ based on iterative compression.

4. A graph G is a *split graph* if $V(G)$ can be partitioned into sets C and I such that C is a clique and I is an independent set. In Split Vertex Deletion problem, given a graph G and an integer k, the task is to check if one can delete at most k vertices from G to obtain a split graph.

   Provide a $\mathcal{O}^*(2^k)$ algorithm for the problem using iterative compression.

5*. In the Cluster Vertex Deletion problem, we are given a graph G and an integer k, and the task is to find a set X of at most k vertices of G such that $G - X$ is a cluster graph (a disjoint union of cliques). Using iterative compression, obtain an algorithm for Cluster Vertex Deletion running in time $\mathcal{O}^*(2^k)$.