1. Point Line Cover

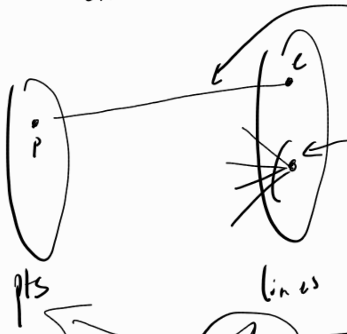
- if a line contains $>k$ (meaning $\geq k+1$) pts, it has to be included in the solution
  - otherwise each of those $>k$ pts has to be covered by a different line but is not good for us

<u>rule 1</u>: If a line ℓ contains $\geq k+1$ pts

- decrease k by 1
- remove all pts on ℓ

- can we try to bound the instance size yet?
- due to repeated application of rule 1, each line contains $\leq k$ pts
- how many lines can we have left? (Ideally a fct of k)

- let us draw an incidence graph between pts and lines



edge exists if $p \in \ell$

deg $\leq k$ by rule 1

- if we have a yes instance, then there can be $\leq k$ lines in the sense that a line is defined by 2 distinct pts

$\rightarrow$ means that part has $\leq k$ vertices

- and each of these vertices has deg $\leq k$ $\rightarrow$ pts part has $\leq k \cdot k = k^2$ vertices

<u>rule 2</u>: After applying rule 1 exhaustively, if there are $>k$ lines (for current k), output no.

- this rule is important because it is not a priori clear that this is true
  - in particular, the instance obtained by only applying rule 1 may be unbounded in size

- rules 1, 2 are clearly applicable in poly(n)

→ we have a $O(k^2)$ kernel

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2. Cluster Editing

G is a cluster gr <=> no P_3 as induced subgr

=>

$K_?$ $K_?$ $K_?$

. every three vertices are either a triangle (if they come from the same component)

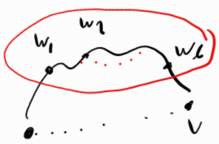
. or it's  if u, v are from the same component and w comes from a different component

. or  if they are all from a different component

<=

. for ∀ suppose that there is a component that is not a clique

. look for the closest (wrt shortest path in an unweighted gr) pair of vertices u v in the component s.t. $uv \notin E$, suppose uv is conn w/ a shortest path w_i of len ≥2



no edges

. as uv is the closest pair, there is no edge between w_i's, otherwise we can shortcut the path

. but what if we move u to w_1? It still holds that $w_1 v \notin E$ → ∀ w/

→ so the path between u and v is actually $u\, w_1\, v$

. looks like induced P_3 to me

. so induced P_3's are a problem, let's try to get rid of them

rule 1: If vertex u is not a part of an induced P_3, delete it as it won't be in the solution.

rule 2: If edge uv is a part of $> k$ induced P_3's, then delete uv, otherwise the solution would have to be too big.

rule 3: If non-edge uv is ——— " ———

———

· now we bound kernel size

· again, there can be at most k disjoint induced P_3's after applying rules 1,2,3 o/w we have a no instance

rule 4: If after applying rules 1,2,3 there are more than k disjoint induced

· now we know that each edge is a part of $\leq k$ induced P_3's | rule 1

——— " ——— each nonedge ——— " ——— | rule 2

every edge is a part of some induced P_3's | rule 3

rule 4: if there's more than k edges after applying rules 1,2,3, output no

· now we bound the size

· an induced P_3 is 2 edges + 1 nonedge $\rightarrow$ $\binom{k}{2}$ of them

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3. d-Bounded Degree Deletion

· same ideas as in Vertex Cover

rule 1: Delete isolated vertices

rule 2: Delete vertices w/ deg $> k+d$ and decrease k.

· now we are left w/ a gr. where every degree is $\leq k+d$ and ≥ 1

· are we done?

· no, e.g. d-regular grs exist for any $|V| >> k$

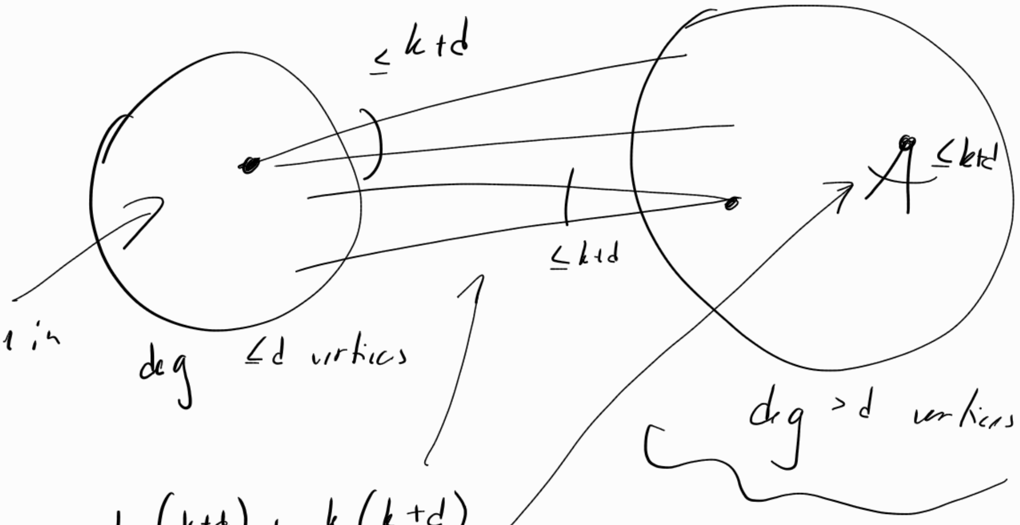
. but we can see that edges between two vertices of deg $\leq d$ do not force us to take either endpt in the solution

rule 3: If $e = uv$ is an edge s.t. $\deg(u) \leq d$ & $\deg(v) \leq d$ $\rightarrow$ delete e

. how about now?

. we know that if there is a vertex of deg $\leq d$, all its neighbors are vertices w/ deg $> d$ & $\leq k+d$ (rule 1)

. actually vertex has deg $\leq k+d$



$\rightarrow$ kernel size $k(k+d) + k(k+d)$

$\leq k$ o/w a sol doesn't exist

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4. RAMSEY

• Ramsey's thm: every graph of size $\geq 4^k$ has a clique or IS of size k

rule 1: If $|V| \geq 4^k$, return YES. Else input is the kernel of size $\leq 4^k$
