2.

Clique <=> IS

· a clique in G is an IS in G

3SAT <=> IS

idea:

· if a fla is satisfiable, then in every clause there is a satisfied literal

· for each clause we add a group of vertices

· the independent set will pick a vertex from each group

· if IS has size # clauses => each clause has a vertex of IS

=> this IS vertex satisfies the clause

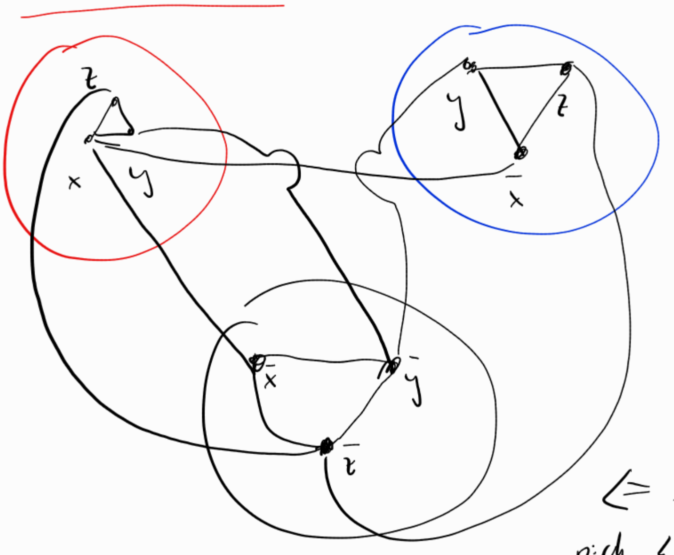
· for clause $C_i = (l_{i1}, l_{i2}, l_{i3})$ we add three vertices u_{i1}, u_{i2}, u_{i3}

and add edges $u_{i1}u_{i2}$, $u_{i2}u_{i3}$, $u_{i3}u_{i1}$

· for two vertices $u_{ia} \in C_i$, $u_{jL} \in C_j$ ($i,j \in [3]$) we add an edge if they represent opposing literals

example.

$\varphi = (x \vee y \vee \bar z) \wedge (\bar x \vee y \vee z) \wedge (\bar x \vee \bar y \vee \bar z)$



Claim: m = # clauses of φ.

φ is satisfiable <=> our gr. has IS of size m

=> If there is a sat assgn, we can pick for each clause the vertex which corresponds to the satisfied literal. Sat assgn has for each var either $x=0$ or $x=1$ => there is no edge between two picked vertices

<= If there is an IS of size m, it can pick ≤ 1 vertex from each clause. These vertices form a valid variable assgn as there are no edges $x\bar{x}$.

IS => VC

. the complement of IS in G is a VC

3.

. let $x_1, \dots, x_n$ be variables

. if φ is not satisfiable => no, done

. for $i \in [n]$:

. if φ w/ $x_i = 1$ is sat:

set $x_i = 1$

else if φ w/ $x_i = 0$ is sat:

set $x_i = 0$

else:

this cannot happen because φ is sat so the sat assign either has

$x_i = 0$ or $x_i = 1$

4.

. it is an FPT alg w/ param $n-k$

. param k? FPT alg probably doesn't exist (you will see later)

5.

. suppose we recurse on u w/ neighbors $u_1, \dots, u_d$

. we can bound the runtime on a n vertex gr. by

$$T(n) \leq 1 + T(n - \deg(v) - 1) + \sum_{i=1}^{d} T(n - \deg(u_i) - 1)$$

. we choose a vertex u w/ min deg => $\deg(u_i) \geq \deg(u)$

. set $s = \deg(u) + 1$

$\rightarrow T(n) \leq 1 + s \cdot T(n-s) \leq 1 + s + s^2 + \dots + s^{n/s} = O^*(s^{n/s})$

• maximized when for integers $s=3 \to O^*(3^{n/3})$ alg.