**Summary of Paging algorithms.** Next to each algorithm name is the description of which page gets evicted when the cache becomes full. We denote the size of the cache by $k$.

- LRU (Least Recently Used) – Page whose *most recent access* happened *furthest* in the past.
- FIFO (First-In First-Out) – Page which was in the memory the *longest.*
- LIFO (Last-In First-Out) – Page which was in the memory the *shortest.*
- LFU (Least Frequently Used) – Page which was requested the *least often.*
- FWF (Flush When Full) – All the pages.
- RAND – An uniformly random page.
- MARK – An *unmarked* page uniformly at random. We require that a cached page becomes marked if it is requested. During the eviction, if all pages are marked, we clear the marks. Thus RAND is a (simplest) marking algorithm.
- LFD (Longest Forward Distance) – Page whose next request is the furthest in the future. This is an *offline* algorithm.

**Deterministic** Paging.

1. What is the time and space complexity of each paging algorithm in the list above? We do not count the space to store the pages themselves.

2. Show that LIFO and LFU are not competitive.

$k$-Server **problem.**

3. Find an algorithm which computes the offline optimum for $k$-Server.

   **Hint.** Reduce to Minimum-Cost Maximum Flow (the arc costs may be negative).

4. Recall the Double Coverage-Tree (DC-T) algorithm for $k$-Server on trees, and the fact that Paging is a special case of $k$-Server on trees. What is the tree for Paging? Which paging algorithm does DC-T correspond to?

**Randomized** Paging.

5. Show that RAND is $k$-competitive.

   We use the following potential
   $$\Phi_i = k(k - x_i),$$
   where $x_i$ is the number of pages common to RAND and the optimum adversary after servicing the $i^{\text{th}}$ request.

   a) How can we use the potential to amortize the cost of RAND? What do we want to show about the amortized cost to show $k$-competitiveness?

   b) Suppose that page $r$ is requested. Depending on whether the adversary has $r$ in their cache or not, analyse the change of potential and the amortized cost.

   c) Can you find an input on which the expected cost of RAND is $k$-times the cost of optimum?