Algorithms for Low Highway Dimension Graphs

Tung Anh Vu

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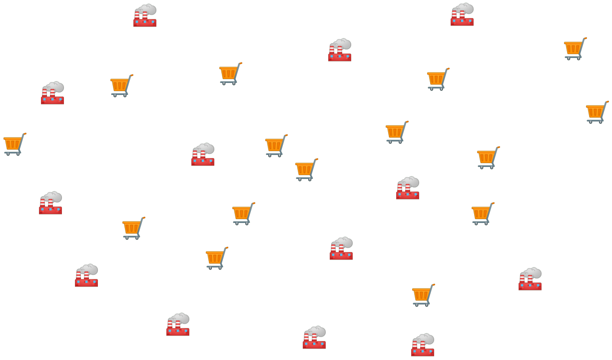
Supervisor: Andreas Emil Feldmann, Dr.

k-Supplier with Outliers

Input

- shortest path metric (V, dist),
- *suppliers* $V_s \subseteq V$ and *clients* $V_c \subseteq V$,
- integers $k, p \in \mathbb{N}_0$.



k-Supplier with Outliers

- $B_u(r) := \{v \in V : \text{dist}(u, v) \leq r\}$.

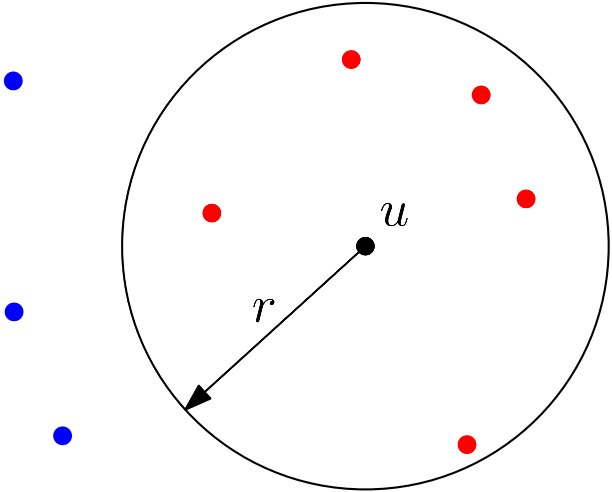


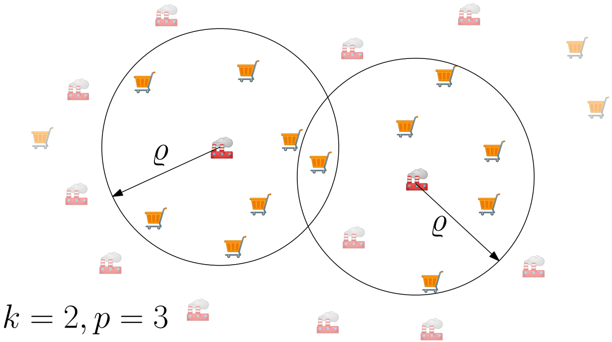
Figure: A ball of radius r centered at vertex u.

k-Supplier with Outliers

Goal

Find a minimum $\varrho \in \mathbb{R}^+$ such that

- there exists $S \subseteq V_s$ of size $|S| \leq k$, and
- $\left| \left(\bigcup_{s \in S} B_s(\varrho) \right) \cap V_c \right| \geq |V_c| - p$.



Parameterized algorithms

Theorem (Hochbaum and Shmoys 1986)

k-SUPPLIER WITH OUTLIERS is NP-hard.

Parameterized algorithms

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k-SUPPLIER WITH OUTLIERS *is* NP*-hard.*

*P*arameterized algorithm

- outputs the exact solution,
- in time $f(q) \cdot n^{O(1)}$ where
  - q is some *parameter* of the input, and
  - f is a computable function.

k-Supplier with Outliers and parameterized algorithms?

Choosing a parameter for k-Supplier with Outliers:

k-Supplier with Outliers and parameterized algorithms?

Choosing a parameter for k-Supplier with Outliers:

- $|V_s| + k + p$?

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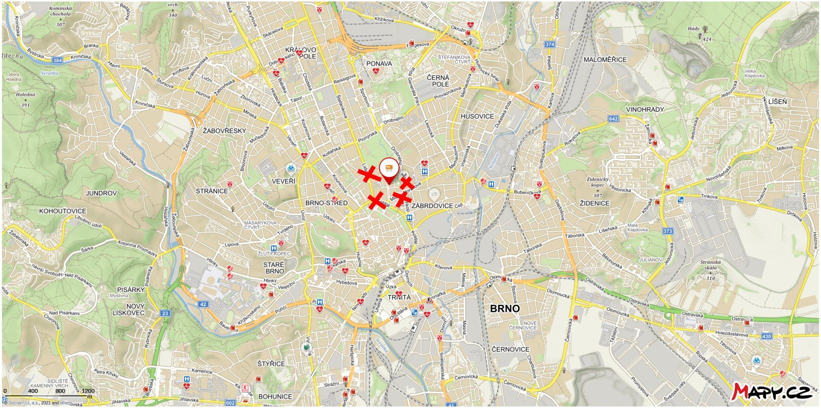
Theorem (Feldmann 2019)

It is $\mathrm{W}[2]$*-hard to* $(2 - \varepsilon)$*-approximate* k*-Center for parameter* k*.*

Parameter for transportation networks: highway dimension



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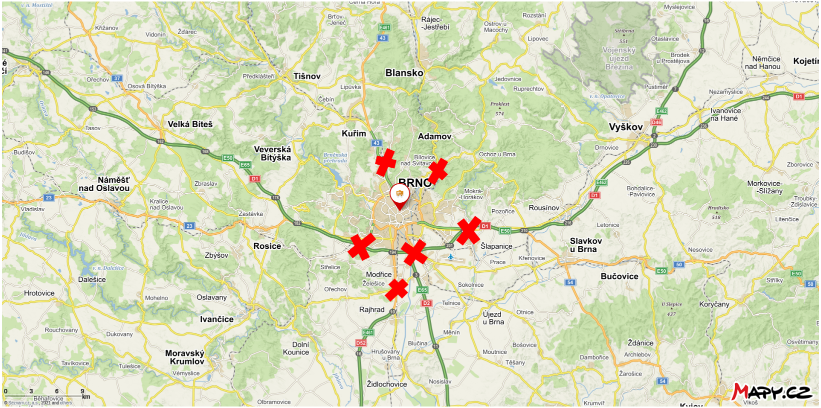

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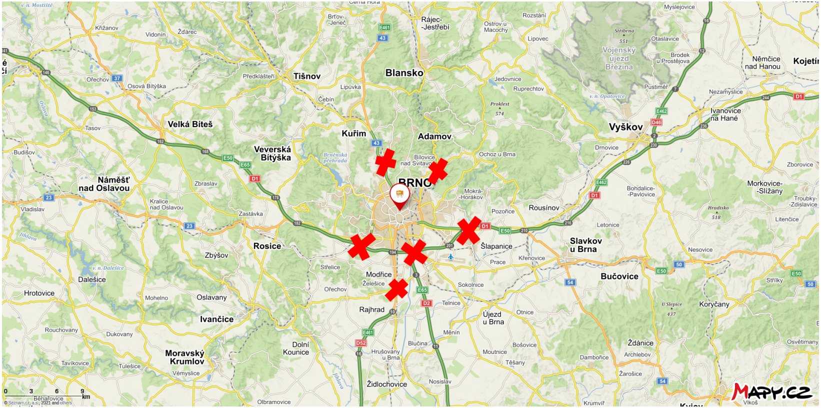

Takeaway

Shortest paths pass through a sparse set of access points.

Parameter for transportation networks: highway dimension

Theorem

For any $\varepsilon > 0$, there exists a $(1+\varepsilon)$-approximation algorithm[1] *for* k-Supplier with Outliers *running in time* $f(h, k, p, \varepsilon) \cdot n^{O(1)}$ *for some computable function* f.

[1] Such an algorithm outputs a solution that is at most $(1+\varepsilon)$ times worse than the optimum.

Parameter for transportation networks: highway dimension

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Question

Is it necessary to approximate?

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Theorem (Feldmann and Marx 2020)

Even on planar graphs of doubling dimension $O(1)$, k-Center is $\mathrm{W}[1]$*-hard for the combined parameter k, pathwidth, and highway dimension.*

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Overview of results

k-SUPPLIER WITH OUTLIERS:

- $\mathrm{tw}^2 + \varepsilon$: $(1+\varepsilon)$-approximation.
- $k + p + \mathrm{hd} + \varepsilon$: $(1+\varepsilon)$-approximation.

CAPACITATED *k*-SUPPLIER WITH OUTLIERS:

- $k + p + \mathrm{dd}^3 + \varepsilon$: $(1+\varepsilon)$-approximation.
- $k + \mathrm{tw}$: $(2-\varepsilon)$-approximation is $\mathrm{W}[1]$-hard.

NON-UNIFORM *k*-SUPPLIER:

- $k + \mathrm{tw} + \mathrm{dd} + \mathrm{hd}$: constant approximation is NP-hard

[2] treewidth

[3] doubling dimension

Thank you for your attention