

Chernoff bounds

Notations

$X_i \in [0, 1]$, $i = 1, \dots, n$ – **independent** random variables

$$X = \sum_{i=1}^n X_i$$

$$\mu = E[X] = \sum_{i=1}^n E[X_i]$$

$$\delta \in (0, 1)$$

Multiplicative error bounds (depending on the mean)

$$Pr[X \geq (1 + \delta)\mu] \leq e^{-\delta^2\mu/3} \quad Pr[X \leq (1 - \delta)\mu] \leq e^{-\delta^2\mu/2}$$

$$Pr[X \geq R] \leq 2^{-R}, \quad R \geq 6\mu$$

Additive error bounds (symmetric)

$$Pr[X \geq \mu + \delta\sqrt{n}], \quad Pr[X \leq \mu - \delta\sqrt{n}] \leq e^{-2\delta^2}$$

$$Pr[X \geq \mu + \delta n], \quad Pr[X \leq \mu - \delta n] \leq e^{-2n\delta^2}$$