

## Homogeneous Structures

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Our purpose is to survey what is known about homogeneous structures over a finite relational language. We will sketch the main results obtained so far, and, by making some conjectures, will identify the most serious gaps in our knowledge. The situation can be summarized as follows: Finite homogeneous structures are well understood. Stable homogeneous structures turn out to be just the unions of chains of finite ones. Thus, understanding stable homogeneous structures goes hand in hand with understanding finite ones. Beyond this, some special cases have been investigated successfully, but almost no general results have been obtained.

The results of the work on special cases consisting of exhaustive lists of the homogeneous structures of various particular kinds, e.g., graphs, are described in §2. In §3 we survey the theory of stable homogeneous structures, and in §4 we speculate on what might be true in general.

**1. Preliminaries.** Let  $L$  be a finite relational language and  $\mathcal{M}$  an  $L$ -structure. Then  $M$  denotes the universe of  $\mathcal{M}$  and  $\text{Th}(\mathcal{M})$  its first-order theory.  $\mathcal{M}$  is *imprimitive* if there is a nontrivial equivalence relation on  $M$  0-definable in  $\mathcal{M}$  (i.e., definable by a formula with no parameters). Otherwise,  $\mathcal{M}$  is *primitive*.

The  $L$ -structure  $\mathcal{M}$  is *homogeneous* if it is countable and any isomorphism between finite substructures extends to an automorphism of  $\mathcal{M}$ . The class of all homogeneous  $L$ -structures is denoted  $\text{Hom}(L)$ .

Let  $\mathbf{F}(L)$  denote the class of all finite  $L$ -structures. A subclass  $\mathbf{U} \subseteq \mathbf{F}(L)$  is called *universal* if it is closed under isomorphism and substructures.  $\mathbf{M}$  is a *constraint* of  $\mathbf{U}$  if all its proper substructures lie in  $\mathbf{U}$  but  $\mathbf{U}$  itself does not. The closure  $\mathbf{U}^c$  of  $\mathbf{U}$  is the class of all  $L$ -structures which are unions of ascending chains of members of  $\mathbf{U}$ . A universal class  $\mathbf{U}$  is *finitely constrained* or *strictly universal* if its class of constraints is finite modulo isomorphism. A class  $\mathbf{A} \subseteq \mathbf{F}(L)$  has the *amalgamation property (AP)* if  $\mathbf{A}$  is universal and for all  $\mathcal{M}, \mathcal{M}_0, \mathcal{M}_1$  in  $\mathbf{A}$  and embeddings  $F_i: \mathcal{M} \rightarrow \mathcal{M}_i$  ( $i = 0, 1$ ) there exist  $\mathcal{N}$  in  $\mathbf{A}$  and embeddings  $G_i: \mathcal{M}_i \rightarrow \mathcal{N}$  such that  $G_0F_0 = G_1F_1$ . Also,  $\mathbf{A} \subseteq \mathbf{F}(L)$  has the *joint embedding property (JEP)* if for all  $\mathcal{M}_0, \mathcal{M}_1$  in  $\mathbf{A}$  there exists  $\mathcal{N}$  in  $\mathbf{A}$  in which both  $\mathcal{M}_0$

and  $\mathcal{M}_1$  are embeddable.  $\mathbf{A} \subseteq \mathbf{F}(L)$  is an *amalgamation class* if  $\mathbf{A}$  is nonempty and has both AP and JEP.

The language  $L$  is called *binary* if all the relation symbols are either unary or binary. An important observation is:

**PROPOSITION.** *Let  $L$  be binary and a finite set  $\mathbf{B}$  of finite  $L$ -structures be given. We can check effectively whether the universal class  $\mathbf{U} \subseteq \mathbf{F}(L)$  constrained by the closure of  $\mathbf{B}$  under isomorphism is an amalgamation class.*

This proposition holds because in checking AP we need only look at one-point amalgamations, i.e., at amalgamations in which  $\mathcal{M}_0$  and  $\mathcal{M}_1$  have only one more element than  $\mathcal{M}$ , and then in checking JEP we need only look at one-point structures  $\mathcal{M}_0$  and  $\mathcal{M}_1$ .

With each  $L$ -structure  $\mathcal{M}$  we associate  $\mathbf{S}(\mathcal{M})$ , the class of all finite  $L$ -structures which are embeddable in  $\mathcal{M}$ . The close relationship between homogeneity, amalgamation classes, and admitting elimination of quantifiers was observed by Fraïssé [F] some thirty years ago and is summarized in the following theorem.

**THEOREM 1.** *Let  $L$  be a finite relational language.*

(1) *For an  $L$ -structure  $\mathcal{M}$ ,  $\mathcal{M}$  is homogeneous iff  $\text{Th}(\mathcal{M})$  admits elimination of quantifiers.*

(2) *If  $\mathbf{A} \subseteq \mathbf{F}(L)$  is an amalgamation class, then there exists  $\mathcal{M} \in \text{Hom}(L)$ , unique up to isomorphism, such that  $\mathbf{A} = \mathbf{S}(\mathcal{M})$ .*


(3) *If  $\mathcal{M} \in \text{Hom}(L)$ , then  $\mathbf{S}(\mathcal{M})$  is an amalgamation class.*

(4)  *$\text{Hom}(L)$  is an elementary class.*

From (2) and (3) we see that the study of homogeneous structures is the same as the study of amalgamation classes. From (1) we deduce that if  $\mathcal{M} \in \text{Hom}(L)$ , then  $\text{Th}(\mathcal{M})$  is  $\aleph_0$ -categorical.

If  $\text{Th}(\mathcal{M})$  is  $\aleph_0$ -categorical, then for each  $n < \omega$ , the subsets of  $M^n$  invariant under  $\text{Aut}(\mathcal{M})$  are the same as those definable without parameters in  $\mathcal{M}$ . Thus, instead of studying  $\mathcal{M}$ , we can study the permutation structure  $(M, \text{Aut}(\mathcal{M}))$ . This seemingly trivial remark turns out to be extremely useful, particularly when dealing with finite structures, because it makes available deep results from the theory of permutation groups. Let  $X$  be a countable set and  $G$  a subgroup of  $\text{Sym}(X)$ , i.e., let  $(X, G)$  be a permutation group of countable degree.  $(X, G)$  is then a *permutation structure* if  $G$  is complete in the topology of pointwise convergence, and for each  $n < \omega$ ,  $X^n$  has finitely many orbits under  $G$ . The permutation structures are just the pairs  $(M, \text{Aut}(\mathcal{M}))$  which arise from countable  $\aleph_0$ -categorical structures. For  $1 \leq k < \omega$ , call  $(X, G)$  *k-ary* if for  $n < \omega$  and tuples  $\bar{a}, \bar{b} \in X^n$  not conjugate under  $G$ , there are corresponding subsequences  $\bar{a}', \bar{b}'$  of  $\bar{a}, \bar{b}$  having length at most  $k$  which are also not conjugate under  $G$ . If  $k$  is the maximum arity of the relation symbols of  $L$ , then the permutation structures arising from structures in  $\text{Hom}(L)$  are *k-ary*. Conversely, every *k-ary* permutation structure arises in this way.

**2. Examples.** Let  $L_0$  be the language with one binary relation symbol  $R$ . Most of the special cases mentioned in the introduction concern the intersection of  $\text{Hom}(L_0)$  with some universal class of  $L_0$ -structures. Below we shall give tables listing the homogeneous graphs, partial orders, tournaments, etc. In each case it is easy to verify that the structures listed are homogeneous, but hard to show that the list is complete.

We can represent  $L_0$ -structures by diagrams in which  $a \circ \rightarrow \circ b$  means “ $(a, b) \in R$  and  $(b, a) \notin R$ ,” and  $a \circ \longleftrightarrow \circ b$  means “ $(a, b), (b, a) \in R$ .” Let  $A, B$ , and  $C$  denote  $\circ \rightarrow \circ$ ,  $\circ \rightarrow \circ \rightarrow \circ$ , and the 3-cycle respectively. Let  $\mathcal{L}$  denote the loop .

Let  $\mathbf{G} \subseteq \mathbf{S}(L_0)$  be the strictly universal class constrained by the structures  $A$  and  $\mathcal{L}$ . Then  $\mathbf{G}^c$  is the class of countable *graphs*. We will list the structures in  $\mathbf{G}^c \cap \text{Hom}(L)$ , which is the same thing as listing the amalgamation classes contained in  $\mathbf{G}$ .

Let  $K_n$  denote the complete graph on  $n$  vertices and  $I_G$  the diagonal of  $G \times G$ . For any graph  $\mathcal{G} = (G, R^{\mathcal{G}})$ , write  $\bar{\mathcal{G}}$  for its *complement*, namely,

$$(G, (G \times G) \setminus (R^{\mathcal{G}} \cup I_G)).$$

The complement of  $K_n$  is denoted  $I_n$ . For any graphs  $\mathcal{G}_0, \mathcal{G}_1$ , let  $\mathcal{G}_0[\mathcal{G}_1]$  be the wreath product obtained by replacing each vertex of  $\mathcal{G}_0$  by a copy of  $\mathcal{G}_1$ , let  $\mathcal{G}_0 \times \mathcal{G}_1$  be the usual cartesian product, and let  $\mathcal{G}_0 + \mathcal{G}_1$  be the disjoint union. Let

$$Pe = (\{0, 1, 2, 3, 4\}, \{(i, j) : |i - j| \in \{1, 4\}\})$$

be the pentagon.

TABLE 1. Homogeneous graphs

Graph $M$	Constraints of $\mathbf{S}(M)$
$Pe$	$K_3, I_3, K_2 \times K_2, \overline{K_2 \times K_2}$
$K_3 \times K_3$	$K_4, I_4, K_1 + K_3, \overline{K_1 + K_3}, K_2 + I_2, \overline{K_2 + I_2}$
$I_m[K_n]$	$I_{m+1}, K_{n+1}, \overline{K_1 + K_2}$
$I_\omega[K_n]$	$K_{n+1}, \overline{K_1 + K_2}$
$I_m[K_\omega]$	$I_{m+1}, \overline{K_1 + K_2}$
$I_\omega[K_\omega]$	$\overline{K_1 + K_2}$
$\mathcal{G}(m)$	$K_{m+1}$
$\mathcal{G}(\omega)$	none

Table 1 lists all homogeneous graphs up to complements;  $m, n$  run through the positive integers. The sets of constraints are relative to  $\mathbf{G}$ , i.e., to each set should be added the constraints of  $\mathbf{G}$ . We have not given explicit constructions of the “generic” graphs  $\mathcal{G}(m)$  and  $\mathcal{G}(\omega)$ . However, from Theorem 1, specifying the amalgamation classes fixes these graphs up to isomorphism. The sources for this table are [G, W2, LW].

Let  $\mathbf{D} \subseteq \mathbf{S}(L_0)$  be the universal class constrained by  $\mathcal{L}$  and  $\mathcal{K}_2$ . Then  $\mathbf{D}^c$  is the class of countable *directed graphs*. For each  $M \in \mathbf{D}$ ,  $\#M$  is obtained by adjoining a new vertex which dominates each vertex of  $M$ , while  $M^\#$  is obtained by adjoining a new vertex dominated by each vertex of  $M$ . Within  $\mathbf{D}$ , let  $\mathbf{T}$  be the universal class constrained by  $I_2$ , and  $\mathbf{P}$  the universal class constrained by  $\mathcal{B}$  and  $\mathcal{C}$ . Then  $\mathbf{T}^c$  and  $\mathbf{P}^c$  are the classes of countable tournaments and partial orders respectively. Let the partial order of the rational numbers be denoted  $\mathcal{Q}$ .

Schmerl [S] established the list of homogeneous partial orders presented in Table 2.

TABLE 2. Homogeneous partial orders

Partial order $\mathcal{M}$	Constraints of $\mathbf{S}(\mathcal{M})$
$I_m$	$\mathcal{A}, I_{m+1}$
$I_m[\mathcal{Q}]$	$\#I_2, I_2^\#, I_{m+1}$
$\mathcal{Q}[I_m]$	$\mathcal{A} + I_1, I_{m+1}$
$I_\omega[\mathcal{Q}]$	$\#I_2, I_2^\#$
$\mathcal{Q}[I_\omega]$	$\mathcal{A} + I_I$
$\mathcal{P}$	none

As before  $1 \leq m < \omega$  and the constraints of  $\mathbf{P}$  have been omitted. The wreath products are defined in the same way as for graphs.  $\mathcal{P}$  is the “generic” partial order. The classifications of homogeneous partial orders by Schmerl, and of homogeneous graphs by Woodrow and the author, confirmed conjectures of Henson [H2]. The most important contribution of Henson’s paper was showing there are  $2^{\aleph_0}$  homogeneous directed graphs; the same was shown by Peretyatkin [P] independently. We will return to this point below.

The author [L3] determined all homogeneous tournaments. They are listed in Table 3.

TABLE 3. Homogeneous tournaments

Tournament $\mathcal{M}$	Constraints of $\mathbf{S}(\mathcal{M})$
$I_1$	$\mathcal{A}$
$\mathcal{C}$	$\mathcal{A}^\#$
$\mathcal{Q}$	$\mathcal{C}$
$\mathcal{Q}^*$	$\mathcal{C}^\#, \# \mathcal{C}$
$\mathcal{T}$	none

The most interesting of these structures is  $\mathcal{Q}^*$ , called a “dense local order” by Cameron [Ca, §6]. The first reference to  $\mathcal{Q}^*$  we know of is [W1, p. 53]. We now come to the recent work of Cherlin [C1, C2], which characterizes all

homogeneous directed graphs except possibly for some primitive ones embedding  $I_\omega$ . Cherlin conjectures that there are at most countably many homogeneous directed graphs not in his catalogue. The following tables summarize Cherlin's work, which subsumes the earlier work on partial orders and tournaments. A directed graph  $M$  is *deficient* if at least one of  $\mathcal{A}$  and  $I_2$  is not embeddable in  $M$ . In Table 4 are listed all the deficient and imprimitive homogeneous directed graphs. Table 5 lists all other *known* homogeneous directed graphs.

TABLE 4. Deficient and imprimitive homogeneous directed graphs

Directed graph $M$	Constraints of $S(M)$
$I_m[\mathcal{X}]$	$\# I_2, I_2^\#, \mathcal{B}, I_{m+1}$ , constraints of $\mathcal{X}$
$I_\omega[\mathcal{X}]$	$\# I_2, I_2^\#, \mathcal{B}$ , constraints of $\mathcal{X}$
$\mathcal{X}[I_m]$	$\mathcal{A} + I_1, \mathcal{B}, I_{m+1}$ , constraints of $\mathcal{X}$
$\mathcal{X}[I_\omega]$	$\mathcal{A} + I_1, \mathcal{B}$ , constraints of $\mathcal{X}$
$\mathcal{Y}^\wedge$	$I_3, \mathcal{A} + I_1, \# Z, Z^\#$ ( $Z$ constraint of $\mathcal{Y}$ )
$m * I_\omega$	$\mathcal{A} + I_1$ , all tournaments of size $m + 1$
$\omega * I_\omega$	$\mathcal{A} + I_1$
$S$	$\mathcal{A} + I_1, \begin{array}{c} \circ \quad \circ \\ \nearrow \quad \searrow \\ \circ \quad \circ \end{array}$

Key :  $\mathcal{X}, \mathcal{Y}$  are homogeneous tournaments,  $\mathcal{Y} \neq \mathcal{Q}^* : 1 \leq m < \omega$ .

The interesting entries in Table 4 are the  $\mathcal{Y}^\wedge$  and the “semigeneric” directed graph  $S$ .  $I_1^\wedge$  and  $C^\wedge$  had turned up previously in [L1].

TABLE 5. Known nondeficient primitive homogeneous directed graphs

Directed graph $M$	Constraints of $S(M)$
$\mathcal{Q}^0$	$C, I_2^\#, \# I_2$
$\mathcal{D}(m)$	$I_{m+1}$
$\mathcal{D}(\mathbf{A})$	$Z$ ( $Z \in \mathbf{A}$ )

Key :  $\mathbf{A}$  is any antichain in  $\mathbf{T}$ ;  $2 \leq m < \omega$ .

As mentioned above Table 5 is only known to be complete with respect to directed graphs not embedding  $I_\omega$ . The surprising entry here is  $\mathcal{Q}^0$ , which Cherlin has called the “myopic local order.” The final entry in Table 5 can be seen as stemming from the discussion in [H2]. Henson showed that there is an infinite antichain  $\mathbf{A} \subseteq \mathbf{T}$ . From this he deduced that there are  $2^{N_0}$  distinct amalgamation classes  $\subseteq \mathbf{D}$  because every subset  $\mathbf{B}$  of  $\mathbf{A}$  constrains a different one. Hence there are  $2^{N_0}$  pairwise nonisomorphic homogeneous directed graphs.

This concludes our account of the progress that has been made in classifying the homogeneous  $L_0$ -structures. Lack of space precludes us from attempting to describe how these results are proved.

**3. Stable homogeneous structures.** An  $L$ -structure  $M$  is *unstable* if for some  $N$  elementarily equivalent to  $M$  there exist  $n < \omega$ ,  $n$ -tuples  $\bar{a}_i \in N$  ( $i < \omega$ ), and an  $L$ -formula  $\varphi(\bar{x}, \bar{y})$  such that  $N \models \varphi(\bar{a}_i, \bar{a}_j)$  iff  $i < j$ . Otherwise,  $M$  is *stable*.

In [CL, Theorem 1] the following theorem is proved by applying the theory of permutation groups. The same result for binary languages was proved earlier in [SL] using purely model-theoretic methods.

**THEOREM 2.** *Let  $L$  be a finite relational language. There is an  $L$ -sentence  $\sigma$  such that for all  $M \in \text{Hom}(L)$ ,  $M$  is stable if and only if  $M \models \sigma$ .*

From [CHL, Corollary 7.4] this theorem is easily seen to be equivalent to:

**THEOREM 2'.** *Let  $L$  be a finite relational language. For all  $M \in \text{Hom}(L)$ ,  $M$  is stable if and only if  $M$  is the union of a chain of finite homogeneous structures.*

*Shrinking.* Consider  $M \in \text{Hom}(L)$ . Let  $\text{Th}(M)$  be  $\aleph_0$ -categorical and  $M \subseteq N$ . Then  $N$  is an *extension by definitions* of  $M$  if  $M$  is 0-definable in  $N$ ,  $\{b\}$  is  $M$ -definable in  $N$  for each  $b \in N$ , and  $\text{Aut}(M) = \{\alpha \mid M: \alpha \in \text{Aut}(N)\}$ . Suppose that in some extension by definitions of  $M$  there is an invariant family  $\Psi$  of pairwise disjoint, definable, infinite indiscernible sets. Let  $\text{Aut}(M)$  act transitively on  $\Psi$ . Suppose further that there is an invariant mapping  $C$  of  $M$  into the finite subsets of  $\bigcup \Psi$  such that for all  $b_0, b_1 \in I \in \Psi$  there exists  $\alpha \in \text{Aut}(M)$  such that  $\alpha(b_0) = b_1, \alpha(C(I)) = C(I)$ , and

$$((\bigcup \Psi) \setminus \{b_0, b_1\}) \cup \{a \in M: b_0, b_1 \notin C(a)\} \subseteq \text{Fix}(\alpha).$$

In this case we call  $\Psi$  a *nice family* attached to  $M$ . Two nice families  $\Psi_0, \Psi_1$  are *equivalent* if there is an invariant bijection between  $\bigcup \Psi_0$  and  $\bigcup \Psi_1$ . The number of inequivalent nice families attached to  $M$  is bounded in terms of  $L$ .

If  $\Psi$  is a nice family attached to  $M$ , we can *shrink  $M$  with respect to  $\Psi$*  as follows. Choose  $m < \omega$ , the *target dimension*, and  $B \subseteq \bigcup \Psi$  such that  $|B \cap I| = m$  for all  $I \in \Psi$ . Let  $N \subseteq M$  be the substructure with universe  $\{a \in M: C(a) \subseteq B\}$ . Then  $N$  is said to be obtained by shrinking  $M$ .  $N \in \text{Hom}(L)$  and is fixed by  $m$  up to an automorphism of  $M$ . There is no difficulty in shrinking  $M$  simultaneously with respect to several inequivalent nice families, each with its own target dimension. More details about shrinking can be found in [L2, §12; L4, and KL].

**EXAMPLE.**  $M = I_\omega[K_\omega]$ . Let  $E$  be the equivalence relation on  $M$  whose classes are the copies of  $K_\omega$ . There are two nice families:  $\Psi_0 = M/E$  and  $\Psi_1 = \{M/E\}$ . Shrinking  $M$  with respect to  $\Psi_0$  gives  $I_\omega[K_m]$ , and with respect to  $\Psi_1$  gives  $I_m[K_\omega]$ .

One of the main results of [L2], Theorem 15.3, says that Theorem 2 above implies:

**THEOREM 3.** *Let  $L$  be a finite relational language. There exists a finite subclass  $\mathbf{H} \subseteq \text{Hom}(L)$  such that, for all  $M \in \text{Hom}(L)$ ,  $M$  is stable iff there exists  $N \cong M$  such that either  $N \in \mathbf{H}$  or  $N$  is obtained by shrinking a member of  $\mathbf{H}$ .*

For the next result we need an additional piece of notation. For any  $L$ -structure  $M$  and  $k < \omega$  let  $\mathbf{U}(M, k)$  denote the class of all finite  $L$ -structures  $\mathcal{F}$  such that every substructure of  $\mathcal{F}$  of size  $\leq k$  is embeddable in  $M$ . The following theorem is due to Harrington and can be used to give a simpler proof of Theorem 3 than is afforded by [L2].

**THEOREM 4.** *Let  $L$  be a finite relational language. There exists  $n$  such that for all stable  $M \in \text{Hom}(L)$ ,  $\mathbf{U}(M, k)$  is an amalgamation class for some  $k < n$ .*

A proof of Harrington's theorem together with the resulting simplified proof of Theorem 2 will appear in [KL]. Combining this theorem with the theory of shrinking and dimensions developed in [L2, §§11 and 12] we immediately obtain the following theorem, which may be thought of as Theorem 4 in another guise. We call an amalgamation class *stable* if the structure associated with it by Theorem 1 is stable.

**THEOREM 5.** *A stable amalgamation class over a finite relational language is finitely constrained.*

**4. Some conjectures.** Fix a finite relational language  $L$  for this section. Let  $\mathbf{A}, \mathbf{B} \subseteq \mathbf{F}(L)$  be finite. We write

$$\bigwedge \mathbf{A} \Rightarrow \bigvee \mathbf{B}$$

if every amalgamation class over  $L$  which includes  $\mathbf{A}$  intersects  $\mathbf{B}$ . Since  $\text{Th}(\text{Hom}(L))$  is axiomatizable, the relation  $\Rightarrow$  is recursively enumerable. One goal of the theory of homogeneous structures is to prove:

**CONJECTURE 1.**  $\Rightarrow$  is recursive.

Note that the theory of stable homogeneous structures sketched ever so lightly in §3 shows that the corresponding relation obtained by restricting to finite amalgamation classes is recursive.

The following is immediate:

**LEMMA.** *The union and intersection, when nonempty, of a chain of amalgamation classes over  $L$  are also amalgamation classes.*

The single most important question about  $\text{Hom}(L)$  is addressed by:

**CONJECTURE 2.** Every amalgamation class over  $L$  is the intersection of a chain of finitely constrained amalgamation classes.

If  $L$  is binary Conjecture 2 implies Conjecture 1. Indeed, the algorithm Conjecture 2 suggests for deciding " $\bigwedge \mathbf{A} \Rightarrow \bigvee \mathbf{B}$ ?" is valid, even if Conjecture 2 fails, provided every pair  $(\mathbf{A}, \mathbf{B})$  which can be separated by an amalgamation class can be separated by a finitely constrained amalgamation class.

Let  $U \subseteq F(L)$  be a strictly universal class. If  $U \neq \emptyset$ , there is certainly a finitely constrained amalgamation class  $A \subseteq U$  corresponding to a one-point structure. Our final piece of speculation is:

CONJECTURE 3. If  $U \subseteq F(L)$  is a nonempty strictly universal class, then the number of maximal finitely constrained amalgamation classes  $\subseteq U$  is finite.

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