Name and Surname:
Pseudonym:

1	2	3	4	5	6

6A. Exam Paper NMAI059 Probability and Statistics 1 – 17.9.2024

On each paper, write the problem number and your surname.

On this paper, you may also write the selected pseudonym under which your results will be published. (Otherwise, they will be published with your initials.) **Submit the assignment as well (it will be available online).**

Do not write more problems on the same page!

You have **150 minutes** to complete the task.

No calculators, counters, mobile phones, etc., are allowed. (Please turn off your phone ringtones in advance.) **Mobile phones must be stored in a closed bag during the entire test.**

If the result contains expressions that are difficult to calculate without a calculator, do not evaluate them: $137 \cdot 173$ is just as good, if not better, an answer than 23701.

Provide detailed justification for all calculations! Even a correct result without justification is almost worthless.

You may use one (handwritten) cheat sheet in A4 format.

After grading the test, a proposed grade (1 to 5) will be emailed to everyone. This can be improved by one grade during the oral part – i.e., a 4 can be improved to a 3, but a 5 means failure in this exam term.

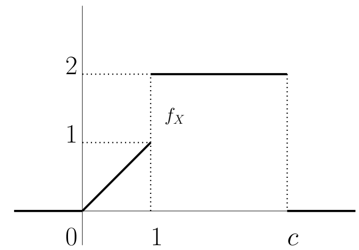
You might find the following table useful.

x	-2.5	-2.0	-1.5	-1.0	-0.5	0.0	0.5	1.0	1.5	2.0	2.5
$\Phi(x)$	0.01	0.02	0.07	0.16	0.31	0.5	0.69	0.84	0.93	0.98	0.99
$\Psi_1(x)$	0.121	0.148	0.187	0.25	0.352	0.5	0.648	0.75	0.813	0.852	0.879
$\Psi_2(x)$	0.065	0.092	0.136	0.211	0.333	0.5	0.667	0.789	0.864	0.908	0.935
$\Psi_9(x)$	0.017	0.038	0.084	0.172	0.315	0.5	0.685	0.828	0.916	0.962	0.983

Provide detailed justification for all calculations! Even a correct result without justification is almost worthless.

1. (10 points) The density of the random variable X is shown in the figure. Outside the marked interval, this function is zero.

(a) Find c for which this is a density.
(b) Is this a discrete or a continuous random variable?
(c) Calculate $P(X < 1/2)$.
(d) What is the median of X?
(e) What is the 80th percentile of X?
(f) Calculate $\mathbb{E}(X)$.



2. (10 points) Each packet transmitted by the local Wi-Fi has a probability $p = 0.98$ of arriving without errors.

(a) What packet will be the first to arrive corrupted? (We are interested in its order in the sequence, more precisely in the expected value of this random variable.)

(b) What packet will be the tenth to arrive corrupted? (Again, in the expected value of its order.)

(c) What is the probability that the first corrupted packet will be the tenth one in the sequence?

3. (10 points) We are measuring the download speed of files from a cloud storage. The download time of each file is a random variable with a mean of $\mu = 5$ minutes and a standard deviation of $\sigma = 2$ minutes. Assume that the download times of individual files are independent, and downloading occurs one after the other (i.e., only one file is downloaded at a time, and immediately after its completion, the next download begins).

(a) If we download 50 files, what is the approximate probability that the total download time exceeds 270 minutes?

(b) What is the approximate probability that the average download time per file is less than 4.5 minutes?

Use the Central Limit Theorem. Write the exact formula using the function Φ and use the table on the previous page for the estimate.

4. (10 points) (a) Define the concept of the density of a random variable.

Determine c for which the function $f(x) = \begin{cases} cx^3, & 0 \le x \le 3 \\ 0, & \text{otherwise} \end{cases}$ is the density of some random variable X. Determine the corresponding distribution function and also $P(1 < X < 2)$.

(b) Define the concept of conditional expectation (only conditioned on an event).

Let D be the outcome of a roll of a twelve-sided die (numbers from 1 to 12). Let T be the event „D is divisible by three“. Calculate $\mathbb{E}(D \mid T)$.

5. (10 points) Explain how to generate random variables. Briefly summarize all the methods we have discussed in class. Explain in detail how inverse sampling works and how to use it to generate a "triangular distribution" (density $f(x) = x/2$ for $x \in [0, 2]$, and $f(x) = 0$ otherwise).

6. (10 points) State the total expectation theorem. Prove it.