Name and Surname:

Pseudonym:

1	2	3	4	5	6

6B. Exam Paper NMAI059 Probability and Statistics 1 – 17.9.2024

On each paper, write the problem number and your surname.

On this paper, you may also write the selected pseudonym under which your results will be published. (Otherwise, they will be published with your initials.) **Submit the assignment as well (it will be available online).**

Do not write more problems on the same page!

You have **150 minutes** to complete the task.

No calculators, counters, mobile phones, etc., are allowed. (Please turn off your phone ringtones in advance.) **During the entire test, mobile phones must be stored in a closed bag.**

If the result contains expressions that are difficult to calculate without a calculator, do not evaluate them: $137 \cdot 173$ is just as good, if not better, an answer than 23701.

Provide detailed justification for all calculations! Even a correct result without justification is almost worthless.

You may use one (handwritten) cheat sheet in A4 format.

After grading the test, a proposed grade (1 to 5) will be emailed to everyone. This can be improved by one grade during the oral part – i.e., a 4 can be improved to a 3, but a 5 means failure in this exam term.


You might find the following table useful.

x	-2.5	-2.0	-1.5	-1.0	-0.5	0.0	0.5	1.0	1.5	2.0	2.5
$\Phi(x)$	0.01	0.02	0.07	0.16	0.31	0.5	0.69	0.84	0.93	0.98	0.99
$\Psi_1(x)$	0.121	0.148	0.187	0.25	0.352	0.5	0.648	0.75	0.813	0.852	0.879
$\Psi_2(x)$	0.065	0.092	0.136	0.211	0.333	0.5	0.667	0.789	0.864	0.908	0.935
$\Psi_9(x)$	0.017	0.038	0.084	0.172	0.315	0.5	0.685	0.828	0.916	0.962	0.983

Provide detailed justification for all calculations! Even a correct result without justification is almost worthless.

1. (10 points) The pdf of a random variable X is shown in the figure. Outside the marked interval, this function is zero.

    (a) Find c for which this is indeed a pdf.
    (b) Is X a discrete or a continuous random variable?
    (c) Calculate $P(X < 1/2)$.
    (d) What is the median of X?
    (e) What is the 90th percentile of X?
    (f) Calculate $\mathbb{E}(X)$.
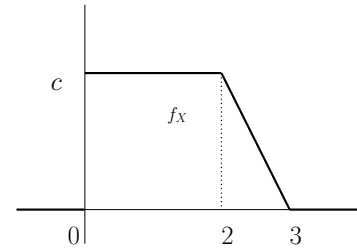
2. (10 points) Each packet transmitted by the local Wi-Fi has a probability of $p = 0.03$ that it will be corrupted due to interference and strong walls.

    (a) What is the probability that all 200 sent packets arrive without errors?
    (b) What is the probability that exactly 6 out of 200 packets will be corrupted?
    (c) Approximate the answer to both previous questions using the Poisson distribution.

3. (10 points) A mobile phone will break (show visible damage) with a probability of p when dropped from a height of one meter. To determine p, we repeatedly drop the phone from this height and measure after how many drops it breaks. We have repeated this experiment a total of five times (i.e., with five pieces of the same type of phone). We have obtained the following data: 5, 4, 5, 6, 6.

    (a) Estimate p using the method of moments.
    (b) Estimate p using the maximum likelihood method.

4. (10 points)   (a) Define the concept of the joint cdf of random variables.
    Let X and Y be discrete random variables with a joint pmf given by:

$$P(X = x, Y = y) = \begin{cases} \dfrac{c}{2^{x+y}}, & x, y = 1, 2 \\ 0, & \text{otherwise} \end{cases}$$

    Find c for which this is a joint pmf. Compute the marginal probabilities and decide whether X and Y are independent.
    (b) Define the concept of correlation of random variables.
    Compute the correlation for the discrete random variables given by the table:

x \\ y	1	3
2	0.3	0.2
4	0.1	0.4

5. (10 points) State the Central Limit Theorem. Explain what it is useful for.
    Using this theorem, estimate the probability that in 180 dice rolls, the number of sixes rolled is at most 25.

6. (10 points) State and prove the convolution formula for discrete random variables (i.e., what is the probability function of their sum).