Name and Surname:

Pseudonym:

1	2	3	4	5	6

4A. Exam Paper NMAI059 Probability and Statistics 1 – 18.6.2024

Write the problem number and your surname on each sheet of paper.

You can also write the selected pseudonym on this paper, under which your results will be published. (Otherwise, they will be published with your initials.) **Submit the assignment as well (it will be available on the web).**

Do not write more problems on the same page!

You have **150 minutes** to complete the work.

No calculators, counters, mobiles, etc. are allowed during the work. (Please turn off the ringtones on your mobiles in advance.)

If the result contains expressions that are difficult to compute without a calculator, do not calculate them: $137 \cdot 173$ is as good, if not better, an answer than 23701.

Provide detailed justification for all calculations.

You can use one (handwritten) cheat sheet in A4 format.

After grading the test, a grade of 1, …, 5 will be suggested to everyone via email. You can improve this grade by one level during the oral part – i.e., a 4 can be improved to 3, but a 5 means failure in this exam term. The oral part of the exam can take place preferably on Friday morning.

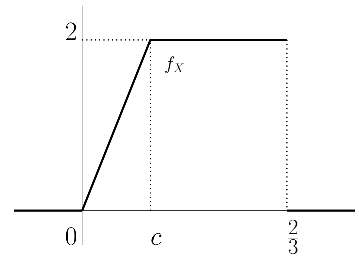
You may find the following table of distribution functions useful.

x	-2.0	-1.5	-1.0	-0.5	0.0	0.5	1.0	1.5	2.0
$\Phi(x)$	0.02	0.07	0.16	0.31	0.5	0.69	0.84	0.93	0.98
$\Psi_1(x)$	0.15	0.19	0.25	0.35	0.5	0.65	0.75	0.81	0.85
$\Psi_2(x)$	0.09	0.14	0.21	0.33	0.5	0.67	0.79	0.86	0.91
$\Psi_9(x)$	0.04	0.08	0.17	0.31	0.5	0.69	0.83	0.92	0.96

Provide detailed justification for all calculations!

1. (10 points) The density of the random variable X is shown in the figure. Outside the indicated interval, this function is zero.

(a) What is the value of c?
(b) What is $P(X < 1/3)$?
(c) What is $P(X > 1/2)$?
(d) What is the sixtieth percentile of X?
(e) Calculate $\mathbb{E}(X)$.



2. (10 points) During a storm, we will model T – the waiting time for the next lightning flash – using an exponential distribution with an expected value of 1 minute.

(a) What is the probability that we will wait more than 2 minutes for the next lightning flash?

(b) What is the probability that the waiting time will be between 9 and 10 seconds?

(c) We measure time with a stopwatch that shows only seconds (i.e., between 9 and 10 seconds it will show 9 seconds), meaning S is T rounded down to seconds. What is the distribution of S? What is the expected value of S?

3. (10 points) 3 million voters are choosing from several parties for the European Parliament. In our (very imperfect) voting model, we will assume that 2 million voters are decided in advance, while 1 million are undecided and will vote randomly.

The party S will receive 200 thousand votes from decided voters. Undecided voters will vote for it with a probability of 10

(a) What is the expected value and variance of the number of votes for party S?

(b) What is the probability that S will get at least 300 300 votes? Express precisely using the distribution function of the appropriate binomial distribution.

(c) Approximate the answer to the previous part using the central limit theorem.

4. (10 points) (a) Define the concept of independent random variables (discrete case, two variables).

Let A, B be two independent rolls of a standard die, M is the greater of them. What is the probability function p_M?

(b) Define the concept of the expected value of a continuous random variable.

For the random variable $X \sim U(0, 1)$, determine $E(X^2 + 3X + 2)$.

5. (10 points) Explain how random variables are generated. In particular, explain how inverse transform sampling works and how to use it to generate the exponential distribution $Exp(\lambda)$.

6. (10 points) State and prove the law of total probability. Define the terms used.