$P_1 =$

$P_2 =$

$P_3 =$

Kombinatorika a grafy III – 2020/21

1.série

1. Describe the following graphs

   - $\mathrm{Forb}_{\subseteq}(C_3, C_4, C_5, \ldots)$ forests
   - $\mathrm{Forb}_{\subseteq}(C_3, C_5, C_7, \ldots)$ bip. graphs
   - $\mathrm{Forb}_{\subseteq}(P_2)$ Each component is a vertex
   - $\mathrm{Forb}_{\subseteq}(P_3)$ -||- or an edge (K_1 or K_2)
   - $\mathrm{Forb}_{\subseteq}(K_{1,n}) = \{G : \Delta(G) \leq n-1\}$
   - $\mathrm{Forb}_{\subseteq}(2K_2)$ = all comp.'s vertices with poss. exception of one; a K_3 or a star $K_{1,t}$

2. Describe the following graphs

   - $\mathrm{Forb}_{\sqsubseteq}(C_3, C_4, C_5, \ldots)$ = forests
   - $\mathrm{Forb}_{\sqsubseteq}(C_3, C_5, C_7, \ldots)$ = bip. graphs (smallest odd cycle is induced)
   - $\mathrm{Forb}_{\sqsubseteq}(P_2)$ = isol. vertices
   - $\mathrm{Forb}_{\sqsubseteq}(P_3)$ = every comp. is a clique

3. ($\star$) Let $\mathcal{G}$ be a $\preceq$-closed class of graphs, where $\preceq$ is a locally finite order. Show that $\mathrm{Obst}_{\preceq}(\mathcal{G}) \subseteq \mathcal{F}$ for every set $\mathcal{F}$ such that $\mathcal{G} = \mathrm{Forb}_{\preceq}(\mathcal{F})$.

4. ($\star$) Describe the graphs in $\mathrm{Forb}_{\subseteq}(P_4)$. $\forall$ comp. is a K_3 or a star $K_{1,t}$

5. ($\star$) Prove that $\mathrm{Forb}_{\sqsubseteq}(C_3, C_5, C_7, \ldots) =$ bipartite.

6. ($\star\star\star$) Describe the graphs in $\mathrm{Forb}_{\sqsubseteq}(2K_2, C_3, C_5, C_7, \ldots)$, that is bipartite graphs without induced matching of size 2.

7. The exact description of $\mathrm{Forb}_{\sqsubseteq}(2K_2)$ is not known.

8. The description of $\mathrm{Forb}_{\sqsubseteq}(K_{1,3})$ (*claw-free graphs*) is known, but it is extremely complicated.

9. For integers a, b we define $\mathcal{G}_{a,b}$ as the class of all graphs having at most a vertices of degree at least b. For which a, b is this class minor-closed? $b \leq 3$

10. Describe $\mathcal{G}_{1,3}$ by forbidden minors.

11. Let G be a connected graph with no $K_{1,k}$-minor. Show that G has at most $10k$ vertices of degree more than 2.

Hint: consider a spanning tree.

a) How many leaves can it have?

b) What is the average deg in any tree?