Combinatorics and graph theory 3 – 2020/21
Series 3

1. Hajós conjecture was that if $\chi(G) \geq k$ then $G \geq_t K_k$. Show that it is false for any k big enough. Hint: it is false for $k \geq 2a_0^2$, where a_0 is such that the Ramsey number $R(a, a) > 2a(a + 1)^2$ for all $a \geq a_0$.

2. Find a graph that is k-connected but not k-linked.

3. Every k-linked graph is $2k - 1$-connected. Must it be $2k$-connected?

4. We say a graph is edge k-linked, if for every $2k$ pairwise distinct vertices $s_1, \ldots, s_k, t_1, \ldots, t_k$ there are *edge-disjoint* paths from s_i to t_i. Show that every edge $2k$-connected graph is edge k-linked.

5. Find edge 2-connected graph that is not edge 2-linked.