

**Combinatorics and Graph Theory III - 2020/21**  
**Series 11**

1. Show how to find number of induced copies of a graph  $F$  in  $G$  using numbers of homomorphisms –  $\text{hom}(F', G)$ , possibly for many graphs  $F'$ .
2. Prove that the Payley graphs from a quasirandom sequence (by verifying condition QR1) .

3. Verify the equality “ $x^2 = x$ ” from class.



4. Translate the „picture proof“ of Goodman’s inequality

$$t(K_3, G) \geq t(K_2, G)(2t(K_2, G) - 1)$$

into a standard proof using the Cauchy-Schwarz inequality twice.

5. Let  $G$  be a simple graph with edge density  $d = t(K_2, G)$ . Prove that

$$t(P_3, G) \leq \max(d^{3/2}, 1 - 2d + d^{3/2}).$$

6. Prove inequality  $t(P_4, G) \geq t(K_2, G)^3$  (by applying Cauchy-Schwarz).

