Kombinatorika a grafy III – 2020/21
1.série

1. Describe the following graphs
   - $\mathrm{Forb}_{\subseteq}(C_3, C_4, C_5, \ldots)$
   - $\mathrm{Forb}_{\subseteq}(C_3, C_5, C_7, \ldots)$
   - $\mathrm{Forb}_{\subseteq}(P_2)$
   - $\mathrm{Forb}_{\subseteq}(P_3)$
   - $\mathrm{Forb}_{\subseteq}(K_{1,n})$
   - $\mathrm{Forb}_{\subseteq}(2K_2)$
2. Describe the following graphs
   - $\mathrm{Forb}_{\sqsubseteq}(C_3, C_4, C_5, \ldots)$
   - $\mathrm{Forb}_{\sqsubseteq}(C_3, C_5, C_7, \ldots)$
   - $\mathrm{Forb}_{\sqsubseteq}(P_2)$
   - $\mathrm{Forb}_{\sqsubseteq}(P_3)$
3. ($\star$) Let $\mathcal{G}$ be a $\preceq$-closed class of graphs, where $\preceq$ is a locally finite order. Show that $\mathrm{Obst}_{\preceq}(\mathcal{G}) \subseteq \mathcal{F}$ for every set $\mathcal{F}$ such that $\mathcal{G} = \mathrm{Forb}_{\preceq}(\mathcal{F})$.
4. ($\star$) Describe the graphs in $\mathrm{Forb}_{\subseteq}(P_4)$.
5. ($\star$) Prove that $\mathrm{Forb}_{\sqsubseteq}(C_3, C_5, C_7, \ldots) =$ bipartite.
6. ($\star\star\star$) Describe the graphs in $\mathrm{Forb}_{\sqsubseteq}(2K_2, C_3, C_5, C_7, \ldots)$, that is bipartite graphs without induced matching of size 2.
7. The exact description of $\mathrm{Forb}_{\sqsubseteq}(2K_2)$ is not known.
8. The description of $\mathrm{Forb}_{\sqsubseteq}(K_{1,3})$ (*claw-free graphs*) is known, but it is extremely complicated.
9. For integers a, b we define $\mathcal{G}_{a,b}$ as the class of all graphs having at most a vertices of degree at least b. For which a, b is this class minor-closed?
10. Describe $\mathcal{G}_{1,3}$ by forbidden minors.
11. Let G be a connected graph with no $K_{1,k}$-minor. Show that G has at most $10k$ vertices of degree more than 2.