

## Flows and cycles in graphs – Exercises 5

1. Let  $G$  be a directed graph with edges colored by red, blue, yellow, and green. Let  $x, y$  be two vertices of  $G$ . We are looking for a path from  $x$  to  $y$  that may use blue edges in the forward direction, yellow in the backward direction, green in both directions and red in neither. Characterize graphs in which there exists such a path (and prove the characterization).
2. A *circular  $r$ -flow* is an  $\mathbb{R}$ -flow that uses only values in interval  $[1, r - 1]$ . Suppose edges of a graph  $G$  can be covered by sets  $A$  and  $B$ , so that  $B$  is a cycle and  $(V(G), A)$  has a circular 2.5-flow. Does  $G$  have a 5-NZF?
3. Try to imitate Seymour's proof of existence of a 6-flow and discuss what goes wrong.