(3.1) $Z=S \dot{\cup} T,|S|=|T|=9 k, Z$ free $\Rightarrow$ a
(3.1) path $P$ from $S$ to $T$ such that no $<k$ cut in $G-V(P)$ nontrivially splits $Z$

large tangle $\mathcal{T}$, free set $Z \Rightarrow$ a set $Y$ free (2.10) w.r.to $\mathcal{T}-Z$ and a matching between $Y$ and $Z$ in $G$
large tangle, nodule $(G, Z), s, t \in V(G)$
$\Rightarrow$ a residual nodule $\left(G^{\prime}, Z^{\prime}\right)$ and a path
(4.2) from $s$ to $t$ in $G-V\left(G^{\prime}\right)$ intersecting
each path of the linkage in at most one
segment
large tangle, nodule $(G, Z) \Rightarrow$ a residual
(4.3) nodule $\left(G^{\prime}, Z^{\prime}\right)$ such that $Z$ belongs to one component of $G-V\left(G^{\prime}\right)$
large tangle, nodule $(G, Z) \Rightarrow$ a nodule
(4.4) $\left(G^{\prime}, Y\right)$ (with $\left.|Y| \gg|Z|\right)$ such that for any $Z^{\prime} \subset Y$ of size $|Z|,\left(G^{\prime}, Z^{\prime}\right)$ is a residual nodule for $(G, Z)$
large tangle and no grid, ordering on vertices of a nodule $\Rightarrow$ a residual nodule
(5.3) + a path joining two non-consecutive paths of the linkage (one of them being among the first few paths)
large tangle and no grid $\Rightarrow$ residual
(5.4) nodule + connected subgraph attaching
to many of paths in its linkage

(5.6) large tangle $\Rightarrow$ large grid or array

