

1. Formulate and prove the variant of the Removal lemma for  $K_4$ .
2. Prove the following strengthening of the Erdős-Stone theorem: Let  $H$  be a graph of chromatic number  $c \geq 2$ . For every  $\beta > 0$ , there exists  $\gamma > 0$  such that every graph  $G$  with  $n$  vertices and at least  $(1 - \frac{1}{c-1} + \beta) \frac{n^2}{2}$  edges contains at least  $\gamma n^{|V(H)|}$  subgraphs isomorphic to  $H$ .
3. Prove that for every  $p > 0$  there exist  $c, \varepsilon > 0$  such that the following claim holds. Let  $G$  be a graph and let  $(A, B)$  be an  $\varepsilon$ -regular pair in  $G$  such that  $d(A, B) \geq p$ . Let  $n = |A| = |B|$ . Suppose  $A' \subseteq A$  and  $B' \subseteq B$  satisfy  $|A'| = |B'| \geq (1 - \varepsilon)n$ , every vertex of  $A'$  has at least  $(p - 2\varepsilon)n$  neighbors in  $B'$ , and every vertex of  $B'$  has at least  $(p - 2\varepsilon)n$  neighbors in  $A'$ . Let  $H$  be the bipartite subgraph of  $G$  with vertex set  $A' \cup B'$  whose edges are exactly the edges of  $G$  with one end in  $A'$  and the other end in  $B'$ . Then  $H$  has at least  $cn$  pairwise edge-disjoint perfect matchings.
4. Prove that for every  $\alpha > 0$ , there exist  $c, n_0 > 0$  such that every graph  $G$  with  $n \geq n_0$  vertices and at least  $\alpha n^2$  edges contains a  $\lceil cn \rceil$ -regular bipartite graph as a subgraph.