1. Let G be a graph and let (A, B) be a (δ, ε)-regular pair in G, where $0 < \delta, \varepsilon \leq 1$. Let $n = |A| = |B|$ and $p = d(A, B)$. Prove that G contains at least $p^4 n^4 - (2\delta + 15\varepsilon)n^4 - 2n^3$ 4-cycles $v_1 v_2 v_3 v_4$ such that $v_1, v_3 \in A$ and $v_2, v_4 \in B$.

2. Let G be a graph and let (A, B), (B, C) and (A, C) be (δ, ε)-regular pairs in G, where $0 < \delta, \varepsilon \leq 1$. Let $n = |A| = |B| = |C|$. Prove that G contains at most $d(A, B)d(B, C)d(A, C)n^3 + (6\delta + 7\varepsilon)n^3$ triangles $v_1 v_2 v_3$ such that $v_1 \in A$, $v_2 \in B$ and $v_3 \in C$.

3. Let $m_0, c, \kappa, \varepsilon, \delta > 0$ be real numbers. Prove that there exists n_0 such that the following holds. Let G be a graph with $n \geq n_0$ vertices and with at most $cn^{2-\kappa}$ edges, and let A and B be disjoint subsets of $V(G)$. If $|A|, |B| \geq n/m_0$, then (A, B) is a (δ, ε)-regular pair in G.

4. Let G be a graph and let (A, B) be a (δ, ε)-regular pair in G, where $0 < \delta, \varepsilon \leq 1$. Let $n = |A| = |B|$ and $p = d(A, B)$. Prove that there exist sets $A' \subseteq A$ and $B' \subseteq B$ such that $|A'| = |B'| \geq (1 - \delta)n$, every vertex of A' has at least $(p - \delta - \varepsilon)n$ neighbors in B' and every vertex of B' has at least $(p - \delta - \varepsilon)n$ neighbors in A'.

5. Let G be a graph and let (A, B) be a (δ, ε)-regular pair in G, where $0 < \delta, \varepsilon \leq 1$. Let $n = |A| = |B|$, let $\beta > \delta$ be a real number, and suppose $A' \subseteq A$ and $B' \subseteq B$ are sets of size at least βn. Prove that (A', B') is a $(\delta/\beta, 2\varepsilon)$-regular pair in G.