1. Prove that every k-linked graph is $(2k-1)$-connected.

2. We say that G is *k-edge-linked* if for every sequence of pairwise distinct vertices $s_1, \ldots, s_k, t_1, \ldots, t_k$, there exist pairwise edge disjoint paths $P_1, \ldots, P_k$ in G, where P_i connects s_i with t_i for $i = 1, \ldots, k$. Prove that every $2k$-edge-connected graph is k-edge-linked.

3. Let $k \geq 2$ be an even integer and let M_k be the cycle on $2k$ vertices where every edge has multiplicity $k/2$. Prove that M_k is k-edge-connected but not k-edge-linked.

4. Let G be a graph, let $K \subseteq G$ be a subdivision of a clique, let Q be the set of branch vertices of K, and let S be a set of m vertices of G. Let $\mathcal{P}$ be a system of m pairwise vertex-disjoint paths from S to Q and let $e_K(\mathcal{P})$ be the number of edges of paths in $\mathcal{P}$ not belonging to $E(K)$; i.e., $e_K(\mathcal{P}) = \left|\left(\bigcup_{P\in\mathcal{P}} E(P)\right) \setminus E(K)\right|$. Choose $\mathcal{P}$ so that $e_K(\mathcal{P})$ is minimum. Prove that if $q \in Q_1$ is not contained in any path of $\mathcal{P}$, $q_2 \in Q$ is contained in a path $P \in \mathcal{P}$, and R is the path of K between q_1 and q_2, then P is the only path from $\mathcal{P}$ that intersects R.

5. Prove using the result from the previous exercise that every $2k$-connected graph containing K_{3k} as a topological minor is k-linked.

6. Find as small integer k as you can for which you can prove that every non-planar k-connected graph is 2-linked.