1. Prove that for every $k \geq 3$, every K_k-minor-free graph can be (not necessarily properly) colored using $k-1$ colors so that every color induces a subgraph of maximum degree less than $3k$.

2. Prove that for all positive real numbers a and $b < 2$ and for every sufficiently large positive integer k, there exists a graph of chromatic number at least ak^b not containing K_k as a topological minor.

3. Prove that for every integer $k \geq 3$, there exists a graph G with at least $\frac{k^2}{16}|V(G)|$ edges not containing K_k as a topological minor (hint: consider complete bipartite graphs).

4. Let G be a 5-connected graph, and let u, v, x, y be distinct vertices of G. Does G necessarily contain vertex-disjoint paths from u to v and from x to y?

5. The $n \times n$ *grid* is the graph with vertex set $\{(i,j) : 1 \leq i,j \leq n\}$ where vertices (i,j) and (i',j') are adjacent iff $i = i'$ and $|j - j'| = 1$, or $j = j'$ and $|i - i'| = 1$. Prove that for every planar graph G, there exists n such that G is a minor of the $n \times n$ grid.