1. Consider $k = 0, 1, 2, 3$. Is a k-sum of two planar graphs necessarily planar?

2. Let G be a connected $K_{1,k}$-minor-free graph. Prove that G contains at most $10k$ vertices of degree at least three (hint: consider a spanning tree of G with the largest number of leaves).

3. For positive integers a and b, let $\mathcal{G}_{a,b}$ be the class of all graphs containing at most a vertices of degree at least b. For which a and b is the class $\mathcal{G}_{a,b}$ closed under minors?

4. Find the set of forbidden minors for $\mathcal{G}_{1,3}$.

5. Prove that every graph G with at least 4 vertices and at least $2|V(G)|-2$ edges contains K_4 as a minor.

6. Prove that if G is a 3-connected graph containing K_5 as a topological minor, then either $G = K_5$ or G contains $K_{3,3}$ as a topological minor. (hint: suppose H is a subdivision of K_5 containing a path $Q = xv_1 \dots v_t y$, where $\deg(x) = \deg(y) = 4$, $\deg(v_1) = \dots = \deg(v_t) = 2$, and $t \geq 1$. If H is a subgraph of G, then since G is 3-connected, it must contain G a path P from $\{v_1, \dots, v_t\}$ to $V(H) \setminus V(Q)$ not containing x and y. Then $H \cup P$ contains a subdivision of $K_{3,3}$).

7. Using the statement of the previous exercise and Kuratowski's theorem, prove that G is $K_{3,3}$-minor-free if and only if G can be obtained from planar graphs and copies of K_5 by (≤ 2)-sums.

8. Using the statement of the previous exercise prove that every graph of minimum degree at least 6 contains $K_{3,3}$ as a minor.