1. A graph G is a *split graph* if we can divide its vertices into two parts A and B such that A is a clique in G and B is an independent set in G. Show that split graphs are perfect.

2. For a graph G, let $\beta(G)$ denote the size of the largest matching in G. Use the Weak Perfect Graph Theorem to show that if G is a bipartite graph, then $\alpha(G) + \beta(G) = |V(G)|$.

3. Prove that every bipartite graph G satisfies $\alpha(G) + \beta(G) = |V(G)|$ directly, using Hall's theorem.

4. Recall that each bipartite graph G satisfies $\chi'(G) = \Delta(G)$; i.e., that linegraphs of bipartite graphs are perfect. What statement about bipartite graphs can we derive from this fact and the Weak Perfect Graph Theorem? The statement should not speak about the properties of the complement of the linegraph of G, but only about the properties of G itself.

5. Show that if G is a cograph, then $P_4 \not\leq G$. Here P_4 denotes the path with four vertices. It may be useful to first determine what is the complement of P_4.

6. Let G be a graph such that both G and the complement of G are connected, and let v be a vertex of G. Show that if $G - v$ has at least two components, then $P_4 \leq G$.

7. Using the previous exercise and induction, show that for every graph G, if $P_4 \not\leq G$, then G is a cograph.