1. Show that for every graph G, we can divide $V(G)$ into two parts A and B so that $\Delta(G[A]) \leq \frac{1}{2}\Delta(G)$ and $\Delta(G[B]) \leq \frac{1}{2}\Delta(G)$.

2. Use the previous exercise and Brook's theorem to prove that every graph of maximum degree at most 7 and clique number at most 3 is 6-colorable.

3. Show that if G is a connected multigraph and G is not an odd cycle, then $\chi'(G) \leq 2\Delta(G) - 2$.

4. Let H be a graph and let v be a vertex of H. Show that if v has three pairwise non-adjacent neighbors, then H is not the linegraph of any multigraph, i.e., that there does not exist any multigraph G such that $L(G) = H$.

5. Show that for every bipartite graph G of maximum degree Δ, there exists a Δ-regular bipartite graph H such that $G \subseteq H$.

6. Use the previous exercise to show that if G is a bipartite graph, then $\chi'(G) = \Delta(G)$.

7. Show that if G is a graph of maximum degree Δ and the vertices of G of degree Δ form an independent set, then $\chi'(G) = \Delta$.