

1. Let Σ be a surface other than the sphere. Show that Σ has a net of minimum degree at least two with only one face, and argue that this implies that Σ has positive Euler genus.
2. Show that every graph drawn in the projective plane is 6-colorable.
3. Let G be a graph drawn in a surface of Euler genus 2. Show that if every proper subgraph of G is 6-colorable, but G is not, then G is 6-regular.
4. Use Brooks' theorem to show that if G is a graph drawn in a surface of Euler genus 2 and $\omega(G) \leq 6$, then G is 6-colorable.
5. Show that every graph drawn in the Klein bottle is 6-colorable.
6. Let G be a graph such that G is not k -colorable, but all proper minors of G are k -colorable. Show that G is 3-connected.