A simple closed curve on a surface is

- *contractible* if it can be continuously shrunk to a point (without breaking the surface), and *non-contractible* otherwise.
- *one-sided* if its small neighborhood is homeomorphic to the Möbius band, and *two-sided* otherwise.
- *separating* if deleting it disconnects the surface, and non-separating otherwise.

Note that every contractible simple closed curve is separating, and that every separating simple closed curve is two-sided.

1. On which of the surfaces of Euler genus at most two (sphere, projective plane, torus, Klein bottle) can we choose a simple closed curve which is
   (a) one-sided?
   (b) non-separating and two-sided?
   (c) non-contractible and separating?
2. Show that the following cyclic strings represent the same surface (up to homeomorphism), for any strings w, w_1, and w_2.
   - $AA^{-1}w$ and w
   - ABw_1ABw_2 and Cw_1Cw_2
   - $Aw_1A^{-1}w_2$ and $Aw_2A^{-1}w_1$
   - Bw_1Bw_2 and $w_1w_2^{-1}CC$ (where $w_2{-}1$ is obtained by reversing the string w_2 and replacing each symbol X by X^{-1} and vice versa).
3. Determine the Euler genus and the orientability of the surface represented by the string $AADBCB^{-1}CD$.
4. For each surface of Euler genus at most two (sphere, projective plane, torus, Klein bottle) determine the largest n such that you can draw K_n on this surface.