The task statements refer to the following definitions.

- For graphs G and H, a vertex $z \in V(G)$, and an edge $uv \in E(H)$, we say that H is obtained from G by *decontracting z to uv* if the graph G is obtained from H by contracting the edge uv to form the vertex z. We say that H is obtained from G by *k-decontracting z* if additionally $\deg_H u, \deg_H v \geq k$.

- For a graph G drawn in the plane, the *rotation system* of the drawing is the function that assigns to each vertex $v \in V(G)$ the cyclic ordering of the edges incident with v in the clockwise order according to how they are drawn around v. The *reflection* of the rotation system is obtained by reversing all the cyclic orderings. We say that two drawings of a graph are *combinatorially equivalent* if they either have the same rotation systems, or the rotation system of one of them is the reflection of the rotation system of the other one.

Exercises:

1. Let G be a k-connected graph with at least $k+1$ vertices, and let H be a graph obtained from G by decontracting a vertex of G to an edge uv of H. Show that the graph H is k-connected if and only if $\deg_H u, \deg_H v \geq k$.

2. Show that a graph with at least four vertices is 3-connected if and only if it can be obtained from K_4 by repeatedly 3-decontracting vertices.

3. Let G be a graph drawn in the plane. Show that G is 2-connected if and only if every face of G is bounded by a cycle.

4. Find a 2-connected planar graph that has two combinatorially non-equivalent drawings in the plane.

5. Show that all drawings of a 3-connected planar graph are combinatorially equivalent (hint: use induction by the number of vertices and the existence of a contractible edge).

6. Show that if a graph G is not 3-connected, then there exist graphs $G_1, G_2 \preceq_m G$ such that $|V(G_1)|, |V(G_2)| < |V(G)|$, $G \subseteq G_1 \cup G_2$, and $G_1 \cap G_2$ is a clique of size at most two (possibly empty one).