1. Prove that if e is a bridge of a graph G, then

$$T_G(x, y) = x \cdot T_{G/e}(x, y).$$

2. Let G be a connected plane graph and let $G^\star$ be its dual. Show that $T_G(x, y) = T_{G^\star}(y, x)$.

3. Show that if G_1 and G_2 are graphs intersecting in at most one vertex, then

$$T_{G_1 \cup G_2}(x, y) = T_{G_1}(x, y) \cdot T_{G_2}(x, y).$$

   Hint: Use induction on $|E(G_1)|$.

4. Compute the Tutte polynomial of the cycle C_n with n vertices.

5. Use the definition of the Tutte polynomial to that for a connected graph G,

   - $T_G(2, 1)$ is the number of acyclic spanning subgraphs of G,
   - $T_G(1, 2)$ is the number of connected spanning subgraphs of G, and
   - $T_G(1, 1)$ is the number of spanning trees of G.

6. Show that $T_G(2, 0)$ is the number of ways how to orient the edges of G so that the resulting directed graph does not contain any directed cycle. Hint: Find a deletion-contraction formula for this quantity.

7. Prove that if e is a bridge of a graph G, then the chromatic polynomial of G satisfies

$$\pi_G(k) = (k - 1) \cdot \pi_{G/e}(k).$$

8. Express the chromatic polynomial in terms of the universal polynomial and in terms of the Tutte polynomial.