

1. Find a perfect matching in the following graphs, or show that they do not have one:

TODO

2. For a graph  $G$ , let  $\mu(G)$  denote the maximum size (number of edges) of a matching in  $G$ . Let  $\nu(G)$  denote the minimum size of a set  $X \subseteq V(G)$  such that every edge of  $G$  has at least one end in  $X$ . Show that  $\nu(G) = |V(G)| - \alpha(G)$  and determine  $\mu(K_n)$ ,  $\nu(K_n)$ ,  $\mu(K_{n,m})$ , and  $\nu(K_{n,m})$ .
3. Show that

$$\mu(G) \leq \nu(G) \leq 2\mu(G)$$

holds for every graph  $G$ . Hint: Consider a largest matching  $M$  in  $G$ . What can you say about the set  $V(G) \setminus V(M)$ ?

4. Let  $G$  be a bipartite graph with parts  $A$  and  $B$  and let

$$r = \max_{S \subseteq A} (|S| - |N(S)|).$$

Show that  $G$  has a matching which covers  $A$  if and only if  $r = 0$ , and that  $\mu(G) = |A| - r$ . Hint: for the second part, add  $r$  new vertices adjacent to all vertices of  $A$  and show that the resulting modified graph has a matching that covers  $A$ .

5. Show that every bipartite graph  $G$  satisfies  $\nu(G) = \mu(G)$ .
6. Describe an algorithm based on maximum flows in networks to compute  $\mu(G)$  for a bipartite graph  $G$ . Show that using this algorithm, you can find a largest independent set in  $G$  in polynomial time.