1. Prove that if a network $(\vec{G}, s, t)$ contains a flow f of non-zero size, then $\vec{G}$ contains a directed path from s to t using only edges e such that $f(e) \neq 0$.

2. Determine the edge-connectivity and the connectivity of the complete bipartite graph $K_{n,m}$.

3. Show that

   - a graph G contains an edge-cut of size at most k if and only if we can partition the vertices of G into two non-empty disjoint parts A and B so that G contains at most k edges with one end in A and the other end in B; and
   - a graph G contains a cut of size at most k if and only if we can partition the vertices of G into three parts A, B, and X, where A and B are non-empty, G does not contain any edges between A and B, and $|X| \leq k$.

4. Show that a connected graph G is 2-edge-connected if and only if every edge of G is contained in a cycle.

5. Show that every graph G satisfies $\kappa(G) \leq \lambda(G)$.

6. Let G be a graph and let u and v be distinct vertices of G. Show that G contains $\kappa(G)$ distinct paths from u to v pairwise vertex-disjoint except for their shared ends. Hint: Apply a theorem from the lecture to obtain a linkage between closed neighborhoods of u and v.