1.4 Tutorial

1. Find explicit formulas for generating functions of the following sequences:
   - 1, 2, 5, 0, 0, 0, ...
   - 1, 1, 1, 1, ...
   - 0, 1, 0, 1, ...
   - 2, −3, 2, −3, ...
   - 0, 1, 2, 3, 4, ... (hint: use derivatives)
   - $0 \cdot 2^0$, $1 \cdot 2^1$, $2 \cdot 2^2$, $3 \cdot 2^3$, $4 \cdot 2^4$, ...
   - 0^2, 1^2, 2^2, 3^2, 4^2, ...
   - 2^2, 3^2, 4^2, 5^2, 6^2, ...
2. Show that if $B(x)$ is the generating function of the sequence b_0, b_1, b_2, ..., then $\frac{B(x)}{1-x}$ is the generating function of the sequence b_0, b_0+b_1, $b_0+b_1+b_2$, ...
3. Use this to derive a formula for the sum $\sum_{k=0}^{n} 3^k$.
4. Suppose e_0, e_1, e_2, ... is a sequence such that $e_0 = 5$, $e_1 = 0$, and $e_n = e_{n-1}+6e_{n-2}$ for every $n \geq 2$. Find an explicit expression for the generating function of this sequence and use it to find an explicit formula for e_n.
5. Denote by t_n the number of ways to cover a $2 \times n$ rectangle with 2×1, 1×2, and 2×2 domino pieces. Determine a linear recurrence and generating function for t_n and use it find an explicit formula.
6. Show that tilings of the $2 \times n$ rectangle are in bijection with tilings of a $1 \times n$ board by one type of length-1 tile and two distinct types of length-2 tiles.