1. For a graph G, let $c_4(G)$ be the number of ordered 4-tuples of distinct vertices (v_1, v_2, v_3, v_4) of G such that $v_1v_2, v_2v_3, v_3v_4, v_1v_4 \in E(G)$. That is, $c_4(G)$ counts the number of 4-cycles in G with a fixed starting vertex and a fixed orientation.

   Let $n, m \geq 4$ be integers such that $m \leq \binom{n}{2}$ and let $p = \frac{m}{\binom{n}{2}}$. Compute and compare the expected values of $c_4(G(n, p))$ and $c_4(G'(n, m))$.

2. Let $p \in (0, 1)$ be a real number. Show that for every $\varepsilon > 0$ there exists $c > 1$ such that the probability that $G(n+1, p)$ contains a vertex of degree less than $(p-\varepsilon)n$ is at most $\frac{n+1}{c^n}$. To solve this exercise, use the inequality given at the end of the tutorial's assignment.

3. Consider the following random process: We start with $2n$ isolated vertices labelled 1, 2, …, $2n$. For $i = 1, 2, \ldots, m$, we choose a pair $\{u_i, v_i\}$ uniformly at random among the $\binom{2(n-(i-1))}{2}$ pairs of vertices belonging to $\{1, 2, \ldots, 2n\} \setminus \{u_1, v_1, \ldots, u_{i-1}, v_{i-1}\}$. Let M be the matching consisting of the edges $\{u_1, v_1\}$, …, $\{u_n, v_n\}$. Show that M is a uniformly random matching; that is, that for every fixed matching M' of $\{1, \ldots, 2n\}$, the probability that $M = M'$ is the same.