1. Show that for every positive integer c, there exists n such that the following claim holds. For any coloring $\varphi : \{1, \ldots, n\} \to \{1, \ldots c\}$ of the first n integers using c colors, there exist numbers $x, y \in \{1, \ldots, n\}$ such that $x + y \leq n$ and $\varphi(x) = \varphi(y) = \varphi(x + y)$. Hint: Consider the complete graph with vertex set $\{1, \ldots, n\}$ and color each edge uv (where $u < v$) by the color $\varphi(v - u)$. Look at a monochromatic triangle in this graph.

2. Show that for every integer $k \geq 3$, there exists an integer n such that the following claim holds: For any coloring φ of edges of the complete graph G with vertex set $\{1, \ldots, n\}$ (using any number of colors), there exists a set $X \subseteq V(G)$ of size k such that either

   - φ assigns the same color to all edges between vertices of X, or
   - there are no vertices $x, y, z \in X$ such that $x < y < z$ and $\varphi(xy) = \varphi(yz)$.

   Hint: Give each triple $\{u, v, w\}$ such that $u < v < w$ a color based on whether $\varphi(uv) = \varphi(vw)$ and apply the Ramsey theorem for triples to obtain a monochromatic set of size $k + 1$.

3. Prove that for every positive integer c and every c-coloring $\varphi : \binom{V}{2} \to \{1, \ldots, c\}$ of 2-tuples of elements of an infinite set V, there exists an infinite monochromatic subset of V. I.e. prove the case $s = 2$ of Theorem 57 from the lecture notes. Hint: Modify the proof of Theorem 53 from the lecture notes.