1. Let $\mathcal{C}$ be the linear code with basis 1000001, 0100010, 0010100, 0001111. Find a check matrix for $\mathcal{C}$, and determine the parameters (length, message length, distance) of this code.

2. Let $\mathcal{C}$ be the linear code with check matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

   Find a basis for $\mathcal{C}$, and determine the parameters (length, message length, distance) of this code.

3. Let $\mathcal{C}_1$ be a Hamming code of length n_1 and let $\mathcal{C}_2$ be a Reed-Solomon code (using a field whose size is a power of two) of length n_2 and minimum distance 3, and let k_1 and k_2 be the message lengths of these codes. Suppose that the parameters of these codes are chosen so that n_1 and n_2 are almost the same, say so that $n \leq n_1, n_2 \leq n + 100$ for a positive integer n. Show that $k_1 \geq k_2 + \Omega(\log n)$; that is, there exists a constant c such that $k_1 \geq k_2 + c \log n$ holds for sufficiently large n.