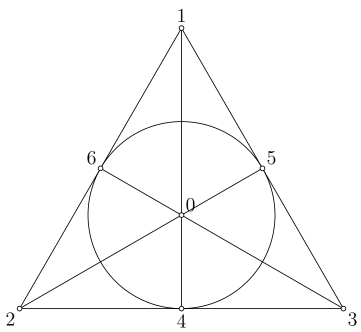
1. Let $\mathcal{C}_1$ be an (n_1, k_1, d_1)-code and $\mathcal{C}_2$ an (n_2, k_2, d_2)-code. Let $\mathcal{C} = \{w_1 w_2 : w_1 \in \mathcal{C}_1, w_2 \in \mathcal{C}_2\}$ be the code whose elements are concatenations of the elements of $\mathcal{C}_1$ and $\mathcal{C}_2$. What are the parameters (length, message length, distance) of $\mathcal{C}$?

2. You are playing the game of "guess a word" with your friend: He chooses one of m possible words, and you can ask him a series of n yes / no questions to determine which word he chose. You are very good at this game, and so for any set W of words, you can formulate a question which has positive answer exactly for the words belonging to W. Your friend is honest and answers the questions correctly, but up to r times, he can refuse to answer the question. Show that if there exists an $(n, k, r+1)$-code such that $m \leq 2^k$, then you have a strategy that allows you to win every time.

3. The *Fano plane* F is the set

$$\{\{1,2,6\},\{2,3,4\},\{1,3,5\},\{0,1,4\},\{0,2,5\},\{0,3,6\},\{4,5,6\}\}$$

(graphically, the elements of F correspond to the sets of points joined by a straight line or a circle in the following picture):



This set has the property that any distinct $x, y \in F$ intersect in exactly one element. Let $\mathcal{C}$ be the set containing

- the strings 0000000 and 1111111, and
- for each $x \in F$, the binary strings o_x and z_x of length 7, where o_x has 1's exactly at positions corresponding to the elements of x and z_x has 0's exactly at these positions. So, for example, for $x = \{1, 2, 6\}$, we have $o_x = 0110001$ and $z_x = 1001110$.

Determine the parameters (length, message length, distance) of the code $\mathcal{C}$.